



Drill Explanations

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1.1 Chapter 2 Solve the Equation Answers

Explanations for Chapter 2 Drill 1 - Solving Equations

	$3x + 2x + 4 = 34$	
1.	$5x + 4 = 34$	Combine like terms $3x + 2x$.
	$5x = 30$	Subtract 4 from both sides.
	$x = 6$	Divide both sides by 5.

	$6x + 12 + x = 3x - 20$	
2.	$7x + 12 = 3x - 20$	Combine like terms $6x + x$.
	$4x + 12 = -20$	Subtract $3x$ from both sides.
	$4x = -32$	Subtract 12 from both sides.
	$x = -8$	Divide both sides by 4.

	$4x + 0.2 = 1.8$	
3.	$4x = 1.6$	Subtract 0.2 from both sides.
	$x = 0.4$	Divide both sides by 4.

	$12x - 1 = x$	
	$-1 = -11x$	Subtract $12x$ from both sides.
	$\frac{1}{11} = x$	Divide both sides by -11.
4.		OR
	$12x - 1 = x$	
	$12x = x + 1$	Add 1 to both sides.
	$11x = 1$	Subtract x from both sides.
	$\frac{1}{11} = x$	Divide both sides by 11.

	$x + 2x + x + 3x - 9 - 12 = 3 + x + x + 1$	
	$7x - 9 - 12 = 3 + 2x + 1$	Combine like terms $x + 2x + x + 3x$ and $x + x$
5.	$7x - 21 = 4 + 2x$	Combine like terms $-9 - 12$ and $3 + 1$.
	$5x - 21 = 4$	Subtract $2x$ from both sides.
	$5x = 25$	Add 21 to both sides.
	$x = 5$	Divide both sides by 5.

1.1 Chapter 2 Solve the Equation Answers and Explanations

6. $5(x+3) = 30$
 $5x + 15 = 30$ Distribute the 5.
 $5x = 15$ Subtract 15 from both sides.
 $x = 3$ Divide both sides by 5.

7. $2(x-12) = 4(x+1)$
 $2x - 24 = 4x + 4$ Distribute the 2 and the 4.
 $-2x - 24 = 4$ Subtract $4x$ from both sides.
 $-2x = 28$ Add 24 to both sides.
 $x = -14$ Divide both sides by -2 .

8. $x - 3(x-5) = 16$
 $x - 3x + 15 = 16$ Distribute the -3 .
 $-2x + 15 = 16$ Combine like terms $x - 3x$.
 $-2x = 1$ Subtract 15 from both sides.
 $x = -0.5$ Divide both sides by -2 .

9. $5(x-12) - 2(2x-16) = 0$
 $5x - 60 - 4x + 32 = 0$ Distribute the 5 and the -2 .
 $x - 60 + 32 = 0$ Combine like terms $5x - 4x$.
 $x - 28 = 0$ Combine like terms $-60 + 32$.
 $x = 28$ Add 28 to both sides.

10. $2.5(x+2) = 2x - 2.5$
 $2.5x + 5 = 2x - 2.5$ Distribute the 2.5 to each term on the left.
 $0.5x + 5 = -2.5$ Subtract $2x$ from both sides.
 $0.5x = -7.5$ Subtract 5 from both sides.
 $x = -15$ Divide both sides by 0.5 (which is the same as multiplying by 2).

Explanations for Chapter 2 Drill 2 - Fractions

1. $\frac{1}{3}x + 2 = 5 - x$
 $3\left(\frac{1}{3}x + 2\right) = 3(5 - x)$ Since we have a denominator of 3, multiply both sides by 3
 $x + 6 = 15 - 3x$ Distribute. $3\left(\frac{1}{3}x\right) = x$.
 $4x + 6 = 15$ Add $3x$ to both sides.
 $4x = 9$ Subtract 6 from both sides.
 $x = \frac{9}{4}$ Divide both sides by 4.

2. $\frac{2}{5} + \frac{1}{15}x = 3$

$15\left(\frac{2}{5} + \frac{1}{15}x\right) = 15(3)$

$6 + x = 45$

$x = 39$

Since we have denominators of 5 and 15, multiply everything by the Least Common Multiple of 5 and 15: 15.

Distribute. $\frac{30}{5} = 6$, and $15\left(\frac{1}{15}x\right) = x$.

Subtract 6 from both sides.

3. $\frac{7}{x} = 140$

$x\left(\frac{7}{x}\right) = x(140)$

$7 = 140x$

$x = \frac{7}{140} = \frac{1}{20} = 0.05$

Since we have a denominator of x , multiply everything by x .

Simplify: $x\left(\frac{7}{x}\right) = 7$.

Divide both sides by 140.

4. $\frac{11}{8} + \frac{3}{2}x = x$

$8\left(\frac{11}{8} + \frac{3}{2}x\right) = 8(x)$

$11 + 12x = 8x$

$11 = -4x$

$x = -\frac{11}{4}$ or -2.75

Since we have denominators of 8 and 2, multiply by the Least Common Multiple of 8 and 2: 8.

Distribute and simplify: $8\left(\frac{11}{8}\right) = 11$, and $8\left(\frac{3}{2}x\right) = 12x$.

Subtract $12x$ from both sides.

Divide both sides by -4 .

5. $\frac{8}{9}x - \frac{5}{9}x = \frac{2}{5} + \frac{4}{15}$

$\frac{3}{9}x = \frac{6}{15} + \frac{4}{15}$

$\frac{1}{3}x = \frac{10}{15}$

$\frac{1}{3}x = \frac{2}{3}$

$x = 2$

Start by simplifying the fractions that are easily simplified. On the left, since we have the same denominator, subtract. On the right, we can put both fractions in terms of 15 by multiplying the first by $\frac{3}{3}$.

Simplify.

Actually, let's simplify just a bit more.

Multiply both sides by 3.

OR

$\frac{8}{9}x - \frac{5}{9}x = \frac{2}{5} + \frac{4}{15}$

$45\left(\frac{8}{9}x - \frac{5}{9}x\right) = 45\left(\frac{2}{5} + \frac{4}{15}\right)$

$40x - 25x = 18 + 12$

$15x = 30$

$x = 2$

The first method works if you happen to see that you can simplify on each side. If you don't, no problem: just do what we've done with the other fraction questions.

The Least Common Multiple of 9, 5, and 15 is 45, so let's multiply everything by 45.

Simplify. It's easier if you reduce your fractions before you multiply:

$$\frac{45(8)}{9} = \frac{5(8)}{1} = 40$$

Combine like terms.

Divide both sides by 15.

1.1 Chapter 2 Solve the Equation Answers and Explanations

$$\frac{3(x-1)}{x+2} = 1 + \frac{2}{x+2}$$

If, instead of having denominators of $x + 2$, this problem had denominators of 5, what would you do? You'd multiply everything by 5, right? So here, we'll do the same thing we did earlier: multiply everything by the denominator $(x + 2)$ to get rid of the fractions.

$$(x+2)\frac{3(x-1)}{x+2} = (x+2)\left(1 + \frac{2}{x+2}\right)$$

Multiply everything by $(x + 2)$

6. $3(x-1) = x+2+2$

Cancel out on the left, distribute on the right. $(x+2)\frac{2}{x+2} = 2$.

$$3x-3 = x+4$$

Distribute and simplify.

$$2x-3 = 4$$

Subtract x from both sides.

$$2x = 7$$

Add 3 to both sides.

$$x = 3.5$$

Divide both sides by 2.

$$\frac{3}{x+3} = \frac{5}{x-2}$$

Fraction equals a fraction? Cross multiply.

7. $3(x-2) = 5(x+3)$

Distribute.

$$-2x-6 = 15$$

Subtract $5x$ from both sides.

$$-2x = 21$$

Add 6 to both sides.

$$x = -10.5$$

Divide both sides by -2 .

$$\frac{8}{x+1} = -\frac{7}{x-5}$$

Since there is a negative in front of the fraction on the right, let's put that negative on the top of the fraction, so we don't forget it's there.

$$\frac{8}{x+1} = \frac{-7}{x-5}$$

Great. Now, we have a fraction equals a fraction, and you know what that means: it is time to cross multiply.

8. $-7(x+1) = 8(x-5)$

Cross! Multiply!

$$-7x-7 = 8x-40$$

Distribute.

$$-15x-7 = -40$$

Subtract $8x$ from both sides.

$$-15x = -33$$

Add 7 to both sides.

$$x = \frac{33}{15} = \frac{11}{5} = 2.2$$

Divide both sides by -15 and simplify.

$$\frac{x}{x+5} = \frac{5}{6}$$

A fraction. Equals. A fraction.

9. $6(x) = 5(x+5)$

Cross multiply.

$$6x = 5x+25$$

Distribute.

$$x = 25$$

Subtract $5x$ from both sides.

	$\frac{3}{x+3} - \frac{9}{2} = 0$	This is almost a fraction equals a fraction, but one of the fractions is on the wrong side.
	$\frac{3}{x+3} = \frac{9}{2}$	So, we can bring it to the other side by adding $\frac{9}{2}$ to both sides. Now we have a fraction equals a fraction.
10.	$9(x+3) = 3(2)$	Cross multiply.
	$9x + 27 = 6$	Distribute and simplify.
	$9x = -21$	Subtract 27 from both sides.
	$x = -\frac{21}{9} = \frac{7}{3} = -2.33\dots$	Divide both sides by 9.

Explanations for Chapter 2 Drill 3 - Suspicious Questions

	If $3x = 20$, what is the value of $12x$?	The question asks for $12x$, and we're given $3x$. Can we get directly from $3x$ to $12x$?
1.	$12x = 4(3x)$	Rewrite.
	$12x = 4(20)$	Substitute.
	$12x = 80$	Simplify.

2.	If $5x - 11 = 14$, what is the value of $5x$?	The question asks for $5x$, and we've got $5x$ right in the equation we're given, so let's just isolate $5x$.
	$5x = 25$	Add 11 to both sides.

3.	If $3w = 12$, what is the value of $5w + 8$?	The question asks for $5w + 8$, but we're given $3w$. Is there any way to get directly from $3w$ to $5w + 8$?
	$w = 4$	Not really, so let's just solve for w . Divide both sides by 3.
	$5(4) + 8$	Substitute in $w = 4$.
	$20 + 8 = 28$	Simplify.

4.	If $5(3x) = 45$, what is the value of $6x + 2$?	The question asks for $6x + 2$, and gives us $5(3x)$. Can we get to $6x + 2$ easily?
	$3x = 9$	That $6x + 2$ is the same as $2(3x) + 2$, so let's just solve for $3x$. Divide both sides by 5.
	$2(3x) + 2 = 2(9) + 2$	Substitute.
	$18 + 2 = 20$	Simplify.
		OR
	$5(3x) = 45$	Didn't see a quick way to get $6x + 2$? No problem, go straight to solving like normal.
	$15x = 45$	Multiply.
	$x = 3$	Divide both sides by 15.
	$6(3) + 2$	Substitute $x = 3$ into $6x + 2$.
	$18 + 2 = 20$	Simplify.

1.1 Chapter 2 Solve the Equation Answers and Explanations

5. If $6x + 10 = 8$, what is the value of $3x + 5$? The question gives us $6x + 10$, but asks for $3x + 10$. Can we get from $6x + 10$ to $3x + 5$?
 $3x + 5 = 4$ Divide both sides by 2.

6. If $12 - x = 16$, what is the value of $5x$? Is there any easy way to get from $12 - x$ to $5x$?
 $-x = 4$ Nope, so just solve for x . Subtract 12 from both sides.
 $x = -4$ Multiply both sides by -1 .
 $5x = 5(-4) = -20$ Substitute.

7. If $3x + 1 = x - 2$, what is the value of $x + 4$? This problem doesn't give us anything that looks like $x + 4$. Let's just solve it normally.
 $2x + 1 = -2$ Subtract x from both sides.
 $2x = -3$ Subtract 1 from both sides.
 $x = -1.5$ Divide both sides by 2.
 $x + 4 = -1.5 + 4 = 2.5$ Substitute.

8. If $4x + 3 = 11$, what is the value of $8x + 6$? The question gives us $4x + 3$, but we need to find $8x + 6$. Can we get from $4x + 3$ to $8x + 6$ in one step?
 $8x + 6 = 2(4x + 3)$ Yup, $8x + 6$ is two times $4x + 3$.
 $2(11) = 22$ Simplify.

9. If $3x = 15$, what is the value of $4x - 7$? Can we get from $3x$ to $4x - 7$ in one step?
 $x = 5$ Nope. Let's just solve for x . Divide both sides by 3.
 $4(5) - 7 = 20 - 7 = 13$ Substitute and simplify.

10. If $4(w - 8) + 6 = w - 8$, what is the value of $w - 8$? The question asks for the value of $w - 8$, and $w - 8$ is right there in the equation. What luck! Just an unbelievable coincidence, really just quite the happy accident.
 $3(w - 8) + 6 = 0$ Subtract $w - 8$ from both sides. $4(w - 8) - (w - 8) = 3(w - 8)$.
 $3(w - 8) = -6$ Subtract 6 from both sides.
 $(w - 8) = -2$ Divide both sides by 2.
 OR
 $4w - 32 + 6 = w - 8$ Didn't see how to simplify easily? Worried about making a mistake subtracting the whole quantity? Not a problem. Solve for w . Start by distributing.
 $4w - 26 = w - 8$ Combine like terms.
 $4w = w + 18$ Add 26 to both sides.
 $3w = 18$ Subtract w from both sides.
 $w = 6$ Divide both sides by 3.
 $w - 8 = 6 - 8 = -2$ Substitute.

Explanations for Chapter 2 Drill 4 - Radicals

-
1. $\sqrt{x+3} = 7$ Let's isolate \sqrt{x} , then we'll solve for x
 $\sqrt{x} = 4$ Subtract 3 from both sides
 $x = 16$ To undo the square root, square both sides
-
2. $\sqrt{x+7} = 12$ Here, the radical equals 12. We can't simplify anything outside of the radical, so let's get rid of it, so we can deal with that +7
 $x+7 = 144$ Square both sides
 $x = 137$ Subtract 7 from both sides
-
3. $\sqrt{5x+1} = 6$ We can't simplify anything outside of the square root, so let's just get rid of that square root, so we can get to all that algebraic goodness inside.
 $5x+1 = 36$ Square both sides.
 $5x = 35$ Subtract 1 from both sides.
 $x = 7$ Divide both sides by 5.
-
4. $\sqrt{x-8} = 0$ Let's get rid of that square root.
 $x-8 = 0$ Square both sides. $0^2 = 0$.
 $x = 8$ Add 8 to both sides.
-
5. $\sqrt{3+3x} = \sqrt{15-x}$ Everything is all locked up in those square roots. Let's get rid of them.
 $3+3x = 15-x$ Square both sides.
 $3x = 12-x$ Subtract 3 from both sides.
 $4x = 12$ Add x to both sides.
 $x = 3$ Divide both sides by 4.
-
6. $x = \sqrt{-2x+15}$ Since the answers are solution sets, rather than solving let's try out either -5 or 3 .
 A) $\{-5\}$
 B) $\{3\}$
 C) $\{-5, 3\}$
 D) There are no real solutions for x .
 $-5 = \sqrt{-2(-5)+15}$ We'll start with -5 . (trying either -5 or 3 first is fine).
 $-5 = \sqrt{10+15}$
 $-5 = \sqrt{25}$
 $-5 = 5$ Nope. That's not true. Eliminate any answer that includes -5 : (A) and (C) are out.
 $3 = \sqrt{-2(3)+15}$ Now try $x = 3$.
 $3 = \sqrt{-6+15}$
 $3 = \sqrt{9}$
 $3 = 3$ That is indeed true, so 3 is a valid solution for x , and the answer is (B).

-
- $x - 4 = \sqrt{2x}$
 A) 2 only
 B) 8 only
 C) -2 and 8
 D) 2 and 8
- $(2) - 4 = \sqrt{2(2)}$
 $-2 = \sqrt{4}$
7. $-2 = 2$
- $(-2) - 4 = \sqrt{2(-2)}$
 $-6 = \sqrt{-4}$
- The only possible values for x are -2 , 2 , or 8 . Since 2 is in half of the answers, it's a good one to try first: if it works, we're down to (A) and (D), if it doesn't work we're down to (B) and (C).
- Substitute in $x = 2$.
- Nope. Not true. Eliminate the answer choices that include 2 : (A) and (D) are gone.
- Since both (B) and (C) include 8 as an answer, we don't need to check it. Let's try $x = -2$
- Weird. We have a square root of a negative number (which doesn't give us a real answer, see Chapter 11: Exponents, Exponential Growth, and Imaginary Numbers on page ?? for more info). Whatever -4 is (it's $2i$, but again that's not super important here), it definitely does not equal -6 . Eliminate (C), and the answer is (B).
-

- $x = \sqrt{6x - 5}$
 A) 1 only
 B) 5 only
 C) 1 and 5
 D) 0, 1, and 5
- $1 = \sqrt{6(1) - 5}$
 $1 = \sqrt{6 - 5}$
 $1 = \sqrt{1}$
 $1 = 1$
8. $5\sqrt{=6(5)-5}$
 $5 = \sqrt{30 - 5}$
 $5 = \sqrt{25}$
 $5 = 5$
- $0 = \sqrt{6(0) - 5}$
 $0 = \sqrt{-5}$
- x could equal 0 , 1 , or 5 . Maybe several of them. But that's it. Let's try out one of the answers: 1 and 5 are in 3 of the answers, so go ahead and start with one of them.
- Trying out $x = 1$ first. (If you want to try $x = 5$ first, that works too.)
- One definitely equals one, so we have, right here, a true statement. Cross out any answer that doesn't include 1 as an answer, which is: just (B). Now we can try either 0 or 5 .
- Trying 5 first. (Trying $x = 0$ here is fine as well.)
- 5 works! Eliminate any answer that doesn't include 5 , which is (A). We still have (C) and (D) left.
- Now let's try $x = 0$. I guess if we had tried this earlier, we could have saved a step? But we had no way of knowing that, sometimes you just have to make your best guess and go with it.
- Well, that's not true. That's not true at all. Eliminate answer (D), and the only answer left is (C).
-

$$x + 1 = \sqrt{3x + 21}$$

- A) $\{-5, -4\}$
 B) $\{-5, 4\}$
 C) $\{-4, 5\}$
 D) $\{5\}$

$$-5 + 1 = \sqrt{3(-5) + 21}$$

We have several options for what we could try for x here: -5 , -4 , 4 , or 5 . Since -5 , -4 , and 5 each show up in two of our answer choices, they're good numbers to try out first: if they work we've eliminated half of our answers, if they don't work we've eliminated the other half.

I'll try -5 first, for the very important mathematical reason that it was the first number I saw and I don't want to stare at the numbers all day, let's just try one and move on.

9.

$$-4 = \sqrt{-15 + 21}$$

$$-4 = \sqrt{6}$$

That's not true, so let's eliminate the answers that include $x = -5$: (A) and (B) are gone. Since both answers remaining include 5 as an answer, we don't need to check it.

$$-4 + 1 = \sqrt{3(-4) + 21}$$

Try $x = -4$ next.

$$-3 = \sqrt{-12 + 21}$$

A square root equaling a negative number. That's not good. We could, by the way, stop here, but if you're not sure...

$$-3 = \sqrt{9}$$

$$-3 = 3$$

Okay, that is definitely wrong. Eliminate (C), and the answer is (D).

$$\sqrt{x + 8} - 3 = x - 1$$

- A) $\{-4\}$
 B) $\{1\}$
 C) $\{-4, 1\}$
 D) $\{-4, 0, 1\}$

Our possible answers are -4 , 0 , or 1 . We can try any of them first. (If one of those numbers showed up in only 2 answers, they'd be good to try first, but since we don't have that, pick whatever.)

10.

$$\sqrt{(-4) + 8} - 3 = (-4) - 1$$

Trying $x = -4$ because -4 is an important number, cloaked in mystery, but also because it's the first number I saw.

$$\sqrt{4} - 3 = -5$$

$$2 - 3 = -5$$

$$-1 = -5$$

Nope. Eliminate (A), (C), and (D). The answer is the only answer left, (B). We got lucky with that first try. Good for us.

1.2 Chapter 3 Algebra Answers

Explanations for Chapter 3 Practice Drill 1 - Distributing and Combining Like Terms

-
1. $(4x + 1) - (5 + x)$
 $4x + 1 - 5 - x$ Distribute the minus sign.
 $3x - 4$ Combine like terms.
-
2. $2(x + 1) + 3(2x - 1)$
 $2x + 2 + 6x - 3$ Distribute the 2 to each term in the first parentheses and the 3 to each term in the second.
 $8x - 1$ Combine like terms.
-
3. $x - (4 - 5x)$
 $x - 4 + 5x$ Distribute the minus sign.
 $6x - 4$ Combine like terms.
-
4. $(x^2 + 8) - (-2x^2 + 3)$
 $x^2 + 8 + 2x^2 - 3$ Distribute the minus sign to each term in the second parentheses.
 $3x^2 + 5$ Combine like terms.
-
5. $(2x^3 + 5x^2) + 4(x^3 - x^2)$
 $2x^3 + 5x^2 + 4x^3 - 4x^2$ Distribute the 4 to each term in the second parentheses.
 $6x^3 + x^2$ Combine like terms.
-
6. $(m^2 + 4) + (m^2 - 4m)$ We need the sum of the expressions, so add them together.
 $2m^2 - 4m + 4$ Combine like terms.
-
7. $(8x^5 + x^3 + 1) + (2x^4 - 5x^3 + 5)$ We need the sum of the expressions, so add them together.
 $8x^5 + 2x^4 - 4x^3 + 6$ Combine like terms.
-
8. $(3xy^2 + 5y + 8x) - (2xy^2 - 5y + x)$
 $3xy^2 + 5y + 8x - 2xy^2 + 5y - x$ Distribute the minus sign to each term in the second parentheses.
 $xy^2 + 10y + 7x$ Combine like terms.
-
9. $x(2 + 4x) + (x^2 - 3x)$
 $2x + 4x^2 + x^2 - 3x$ Distribute the x to each term in the first parentheses.
 $5x^2 - x$ Combine like terms.
-

-
10. $3a(a+2+b) + (5+b)$
 $3a^2 + 6a + 3ab + 5 + b$
 $3a^2 + 3ab + 6a + b + 5$
- Distribute the $3a$ to each term in the first parentheses.
Rearranged in terms of degree (not necessary, just the way the SAT normally arranges terms in answer choices.)

Explanations for Chapter 3 Practice Drill 2 - FOIL

1. $(x+3)(x+5)$
 $x^2 + 5x + 3x + 15$
 $x^2 + 8x + 15$
- FOIL.
Combine like terms.
-
2. $(x-6)(x+1)$
 $x^2 + x - 6x - 6$
 $x^2 - 5x - 6$
- FOIL.
Combine like terms.
-
3. $(x-7)(x-11)$
 $x^2 - 11x - 7x + 77$
 $x^2 - 18x + 77$
- FOIL.
Combine like terms.
-
4. $(a+3b)(a-5b)$
 $a^2 - 5ab + 3ab - 15b^2$
 $a^2 - 2ab - 15b^2$
- FOIL.
Combine like terms.
-
5. $(x+4)^2$
 $(x+4)(x+4)$
 $x^2 + 4x + 4x + 16$
 $x^2 + 8x + 16$
- Write out the squared terms to make it easier to FOIL.
FOIL.
Combine like terms.
-
6. $(x-8)^2 + 10$
 $(x-8)(x-8) + 10$
 $x^2 - 8x - 8x + 64 + 10$
 $x^2 - 16x + 74$
- Write out the squared terms to make it easier to FOIL.
FOIL.
Combine like terms.
-
7. $(3x-5)(8x+1)$
 $24x^2 + 3x - 40x - 5$
 $24x^2 - 37x - 5$
- FOIL.
Combine like terms.
-
8. $(ab+c)(b+ac)$
 $ab^2 + a^2bc + bc + ac^2$
- FOIL. There are no like terms to combine, so we're done.

9. $(a^5 - b^5)(a^3 + 5b)$
 $a^8 + 5a^5b - a^3b^5 - 5b^6$ FOIL. There are no like terms to combine, so that's it, that's the answer.

10. $\left(x + \frac{y}{2}\right)^2$
 $\left(x + \frac{y}{2}\right)\left(x + \frac{y}{2}\right)$ Write out the squared terms to make it easier to FOIL.
 $x^2 + \frac{xy}{2} + \frac{xy}{2} + \frac{y^2}{4}$ FOIL.
 $x^2 + \frac{2xy}{2} + \frac{y^2}{4}$ Combine like terms.
 $x^2 + xy + \frac{y^2}{4}$ Simplify.

Explanations for Chapter 3 Practice Drill 3 - Making Up Your Own Number

1. $7(2) = 2y$ Plug in $x = 2$.
 $y = 7$ Solve for y .
 $14y = 14(7) = 98$ The question asks for the value of $14y$, so find that. Now we have to check each answer to see which one gives 98.
 A) 2 Nope. Eliminate (A).
 B) $4(2) = 8$ Not 98, so eliminate (B).
 C) $14(2) = 28$ Also not 98, so eliminate (C).
 D) $49(2) = 98$ That's the answer we were looking for, so the answer is (D).

2. $2x = 5 + 2$ Make up a number for either m or x . I made up $m = 2$ because there were a lot of m 's on the equation, and I didn't feel like solving for m .
 $2x = 7$
 $x = 3.5$ Divide both sides by 2.
 $3.5 - 1 = 2.5$ The question asks which one is equivalent to $x - 1$. Now let's find an answer that gives us 2.5.
 A) 5 5 is not the same as 2.5, so eliminate (A).
 B) $5(2) = 10$ Not 2.5, eliminate (B).
 C) $\frac{5}{2} = 2.5$ That works, so keep (C) for now.
 D) $\frac{2}{5} = 0.4$ Nope, eliminate (D), and the answer is (C).

3. $\frac{3}{2(2)} - \frac{5}{8(2)}$
 $\frac{3}{4} - \frac{5}{16}$
 $\frac{12}{16} - \frac{5}{16}$
 $\frac{7}{16}$
- Do we have any reason not to make $x = 2$? No? Then let's make $x = 2$.
- We want a common denominator, so multiply the first fraction by $\frac{4}{4}$.
- Okay, we have our answer. Now let's check each answer choice to see which one gives us $\frac{7}{16}$.
- A) $\frac{17}{8(2)} = \frac{17}{16}$
 B) $\frac{7}{8(2)} = \frac{7}{16}$
 C) $-\frac{17}{8(2)}$
 D) $-\frac{7}{8(2)}$
- That isn't what we wanted. No. Eliminate (A).
- Okay, this works. This is good. Keep (B).
- This is negative. Gone. Eliminate (C).
- Also negative, also gone. Eliminate (D). The answer is (B).

4. $\frac{2^2 + 4(2) + 5}{2 + 3}$
 $\frac{4 + 8 + 5}{5}$
 $\frac{17}{5} = 3\frac{2}{5}$
- Once again, since we have no reason not to, $x = 2$.
- Simplify. I've rewritten $\frac{17}{5}$ as a mixed number, so I can compare it to the answer choices a bit more easily. Check our answers.
- A) 2
 B) $2 + \frac{2}{2+3} = 2 + \frac{2}{5}$
 C) $2 + 1 + \frac{2}{2+3} = 3 + \frac{2}{5}$
 D) $2 + 3 + \frac{2}{2+3} = 5 + \frac{2}{5}$
- That is not what we wanted. Eliminate (A).
- Nope. Eliminate (B).
- That works, keep (C).
- Nope, eliminate (D), and the answer is (C).

5. $5a + 15(2) = 20c$
 $5a + 30 = 20c$
 $5a + 30 = 60$
 $5a = 30$
 $a = 6$
- I decided to make $b = 2$, because it seemed like multiplying 15 by a smaller number would be good.
- I still can't solve for a or c , so let's make up another variable. $5a + 30 = 20(3)$
 I'll say $c = 3$. Again, just trying to keep the numbers small.
- Subtract 30 from both sides.
- Now we have all 3 variables: $a = 6$, $b = 2$, and $c = 3$. But we still haven't answered the question yet. What were they asking for?
- $a^2 + 6ab + 9b^2 = 6^2 + 6(6)(2) + 9(2)^2$
 $36 + 36(2) + 9(4)$
 $36 + 72 + 36 = 144$
- Plug in $a = 6$ and $b = 2$.
- Okay, so we want to find out which answer gives us 144.
- A) $4(3) = 12$
 B) $2(3)^2 = 2(9) = 18$
 C) $4(3^2) = 4(9) = 36$
 D) $16(3^2) = 16(9) = 144$
- That's not 144, eliminate (A).
- Also not 144, eliminate (B).
- (C) is gone.
- That works, so the answer is (D).

Explanations for Chapter 3 Practice Drill 4 - Balance the Coefficients, Infinite Solutions

- | | | |
|-------|---|--|
| 1. | $a + 4 + 12x = 3(4x + 5)$
$a + 4 + 12x = 12x + 15$
$a + 4 = 15$
$a = 11$ | <p>For the equation to be true for all values of x, the coefficients must balance out.</p> <p>Distribute the 3 to each term on the right side.</p> <p>The x terms already balance out, so let's look at the constant terms, without an x. They also have to equal.</p> <p>Subtract 4 from both sides.</p> |
| <hr/> | | |
| 2. | $2(x + b) = ax + 10$
$2x + 2b = ax + 10$
$2x = ax$
$2b = 10$
$b = 5$ | <p>For the equation to be true for all values of x, the coefficients must balance out.</p> <p>Distribute the 2 to each term on the left side.</p> <p>The x terms have to balance out, so $a = 2$.</p> <p>The constant terms, without an x, also have to balance out.</p> <p>Divide both sides by 2.</p> |
| <hr/> | | |
| 3. | $\frac{ax + b}{3} = 6x + 12$
$ax + b = 18x + 36$
$ax = 18x$
$b = 36$ | <p>For the equation to be true for all values of x, the coefficients must balance out.</p> <p>Multiply both sides by 3 to get rid of the fraction on the left.</p> <p>The x terms have to balance out, so $a = 18$.</p> <p>The constant terms, without an x, also have to balance out.</p> |
| <hr/> | | |
| 4. | $(x + 5)^2 = ax^2 + bx + c$
$(x + 5)(x + 5) = ax^2 + bx + c$
$x^2 + 5x + 5x + 25 = ax^2 + bx + c$
$x^2 + 10x + 25 = ax^2 + bx + c$
$x^2 = ax^2$
$10x = bx$
$25 = c$ | <p>For the equation to be true for all values of x, the coefficients must balance out.</p> <p>Write out the squared parentheses to make it easier to FOIL.</p> <p>FOIL.</p> <p>Combine like terms.</p> <p>The x^2 terms have to balance out, so $a = 1$.</p> <p>The x terms have to balance out, so $b = 10$.</p> <p>The constant terms, without an x, also have to balance out, so $c = 25$.</p> |
| <hr/> | | |
| 5. | $x(3x + 5) - 2(x - 4) = ax^2 + bx + c$
$3x^2 + 5x - 2x + 8 = ax^2 + bx + c$
$3x^2 + 3x + 8 = ax^2 + bx + c$ | <p>For the equation to be true for all values of x, the coefficients must balance out.</p> <p>Distribute the x in the first parentheses and the -2 in the second.</p> <p>Combine like terms. Compare matching coefficients: $a = 3$, $b = 3$, and $c = 8$</p> |
| <hr/> | | |
| 6. | $3x(2x^4 + 7) = ax^c + bx$
$6x^5 + 21x = ax^c + bx$ | <p>For the equation to be true for all values of x, the coefficients must balance out.</p> <p>Distribute the $3x$. $a = 6$, $c = 5$, and $b = 21$.</p> |
| <hr/> | | |
| 7. | $(2x + 5)(2x - 5) = ax^2 + bx + c$
$4x^2 - 10x + 10x - 25 = ax^2 + bx + c$
$4x^2 - 25 = ax^2 + bx + c$ | <p>For the equation to be true for all values of x, the coefficients must balance out.</p> <p>FOIL.</p> <p>Combine like terms. $a = 4$, $b = 0$, and $c = -25$.</p> |
-

-
8. $(x+2a)(bx+1) = x^2 + 7x + c$ For the equation to be true for all values of x , the coefficients must balance out.
- $bx^2 + x + 2abx + 2a = x^2 + 7x + c$ FOIL. Now we know that $b = 1$, since the x^2 terms have to balance out.
- $1x^2 + x + 2a(1)x + 2a = x^2 + 7x + c$ Insert $b = 1$.
- $2ax + x = 7x$ The x terms also have to balance out.
- $2a + 1 = 7$ Balance the coefficients for the x terms.
- $2a = 6$ Subtract 1 from both sides.
- $a = 3$ Divide both sides by 2.
- $x^2 + x + 2(3)x + 2(3) = x^2 + 7x + c$ Insert $a = 3$.
- $x^2 + x + 6x + 6 = x^2 + 7x + c$
- $x^2 + 7x + 6 = x^2 + 7x + c$ The constant terms, without an x , also have to balance out, and $c = 6$.
-

9. $(4x+a)^2 = bx^2 + bx + c$ For the equation to be true for all values of x , the coefficients must balance out.
- $(4x+a)(4x+a) = bx^2 + bx + c$ Write out the squared parentheses to make it easier to FOIL.
- $16x^2 + 4ax + 4ax + a^2 = bx^2 + bx + c$ FOIL.
- $16x^2 + 8ax + a^2 = bx^2 + bx + c$ Combine like terms. $b = 16$.
- $16x^2 + 8ax + a^2 = 16x^2 + 16x + c$ Insert $b = 16$.
- $8ax = 16x$ The x terms have to balance out.
- $8a = 16$
- $a = 2$ Divide both sides by 8.
- $16x^2 + 8(2)x + (2)^2 = 16x^2 + 16x + c$ Insert $a = 2$.
- $16x^2 + 16x + 4 = 16x^2 + 16x + c$ The constant terms, without an x , also have to balance out, and $c = 4$.
-

10. $\frac{15x^2 + 13x - 8}{ax - 1} = -3x - 2 - \frac{10}{ax - 1}$ For the equation to be true for all values of x , the coefficients must balance out. That means that, after dividing by $ax - 1$, each term on the left should match with a term on the right. The $13x$ term, for instance, will be divided by $ax - 1$ and match with a constant term, because the x 's will cancel out.
- $\frac{15x^2}{ax} = -3x$ Let's just focus on the x terms. On the left side, that's the term with an x^2 , because, after dividing by x , we'll be left with something times x .
- $\frac{15}{a} = -3$ Isolate just the coefficients, so we can balance out each side.
- $15 = -3a$ Multiply both sides by a .
- $-5 = a$ Divide both sides by -3 .
-

Explanations for Chapter 3 Practice Drill 5 - Balance the Equations, No Solutions

1.2 Chapter 3 Algebraic Manipulation Answers and Explanations

1. $k(x+5) = 8(3x+1)$ If the equation has no solutions, all the x terms have to cancel out.
 $kx + 5k = 24x + 8$ Distribute.
 $kx = 24x$ Balance out the coefficients for the x terms. $k = 24$
 $24x + 5(24) = 24x + 8$
 $120 = 8$ That is never true, no matter what x is, so $k = 24$ will make the equation have no solutions.
-
2. $kx^2 = 10$ If the equation has no solutions, all the x^2 terms have to cancel out. There are no x^2 terms on the right, so there should be no x^2 terms on the left
 $0x^2 = 10$
 $0 = 10$ That's not right. That's never true. Never. So if $k = 0$ then the equation will have no solutions.
-
3. $k(x-1) - k(3x+5) = 8x-3$ If the equation has no solutions, all the x terms have to cancel out.
 $kx - k - 3kx - 5k = 8x - 3$ Distribute.
 $-2kx - 6k = 8x - 3$ Combine like terms.
 $-2kx = 8x$ Let's just compare the x terms, to balance those out.
 $-2k = 8$
 $k = -4$ Divide both sides by -2 .
 $-2(-4)x - 6(-4) = 8x - 3$ Using $k = -4$ to prove that this leaves us with no solutions (not necessary on the test, but just here for the sake of completeness).
 $8x + 10 = 8x - 3$
 $10 = -3$ That's never true, no matter what x is.
-
4. $\frac{1}{5}(2x+k) = k(x+1)$ If the equation has no solutions, all the x terms have to cancel out.
 $\frac{2x}{5} + \frac{k}{5} = kx + k$ Distribute the $\frac{1}{5}$.
 $\frac{2x}{5} = kx$ The x terms have to balance out.
 $\frac{2}{5} = k$
-
5. $k(x+2)(x-1) = x^2 + x + 1$ If the equation has no solutions, all the x and x^2 terms have to cancel out.
 $k(x^2 - x + 2x - 2) = x^2 + x + 1$ FOIL.
 $k(x^2 + x - 2) = x^2 + x + 1$ Combine like terms.
 $kx^2 + kx - 2k = x^2 + x + 1$ Distribute the k .
 $kx^2 = x^2$ The x^2 terms should balance out, so $k = 1$.
 $kx = x$ The x terms should also balance out, so you could use those terms as well, which would give you the same answer: $k = 1$.

Explanations for Chapter 3 Practice Drill 6 - Isolate the Variable

-
1. $C = 2\pi r$ in terms of r
 $\frac{C}{2\pi} = r$
- Let's get rid of everything on the right side except for the r . Right now the r term is multiplied by 2π .
 Divide both sides by 2π .
-
2. $E = mc^2$ in terms of m
 $\frac{E}{c^2} = m$
- Let's get rid of everything on the right side except for the m . Right now the m is multiplied by c^2 .
 Divide both sides by c^2 .
-
3. $D = \frac{M}{V}$ in terms of V
 $DV = M$
 $V = \frac{M}{D}$
- We want to get V by itself. Right now V is on the bottom of the fraction, which is a little annoying.
 Multiply both sides by V to get it out of the denominator of the fraction. Now V is multiplied by D .
 Divide both sides by D .
-
4. $F = \frac{9}{5}C + 32$ in terms of C
 $F - 32 = \frac{9}{5}C$
 $\frac{5}{9}(F - 32) = C$
- Let's get C by itself. We'll basically do PEMDAS in reverse, getting rid of any addition or subtraction first.
 Subtract 32 from both sides. Now C is still multiplied by $\frac{9}{5}$, so we need to get rid of that.
 Multiply both sides by $\frac{5}{9}$.
-
5. $K = \frac{1}{2}mv^2$ in terms of m
 $2K = mv^2$
 $\frac{2K}{v^2} = m$
- To get m by itself, we need to get rid of the $\frac{1}{2}$ and the v^2 terms.
 Multiply both sides by 2 to get rid of the $\frac{1}{2}$.
 Divide both sides by v^2 .
-
6. $K = \frac{1}{2}mv^2$ in terms of v
 $2K = mv^2$
 $\frac{2K}{m} = v^2$
 $\sqrt{\frac{2K}{m}} = v$
- To get v by itself, we need to get rid of the $\frac{1}{2}$ and the m terms. We'll also have to get rid of the squared part of that v^2 , but we'll do that later.
 Multiply both sides by 2 to get rid of the $\frac{1}{2}$.
 Divide both sides by m .
 To get rid of the squared part of v^2 , take the square root of both sides.
-
7. $F = G\frac{m_a m_b}{r^2}$ in terms of r
 $Fr^2 = Gm_a m_b$
 $r^2 = \frac{Gm_a m_b}{F}$
 $r = \sqrt{\frac{Gm_a m_b}{F}}$
- Since r is in the denominator, let's start by getting rid of that.
 Multiply both sides by r^2 .
 Divide both sides by F .
 Take the square root of both sides.
-

-
8. $S = 2\pi rh + 2\pi r^2$ in terms of h
- We need to get h by itself, but let's start by getting rid of that entire $2\pi r^2$ part.
- $S - 2\pi r^2 = 2\pi rh$
- Subtract $2\pi r^2$ from both sides. Now we need to get rid of the $2\pi r$ part.
- $\frac{S - 2\pi r^2}{2\pi r} = h$
- Divide both sides by $2\pi r$.
-
9. $V = \frac{4}{3}\pi r^3$ in terms of r
- Let's isolate r by getting rid of the $\frac{4}{3}\pi$.
- $\frac{3}{4}V = \pi r^3$
- Start by multiplying both sides by $\frac{3}{4}$.
- $\frac{3V}{4\pi} = r^3$
- Divide both sides by π . Now we have to get rid of that cube on the r^3 term.
- $\sqrt[3]{\frac{3V}{4\pi}} = r$
- Take the cube root of both sides.
-
10. $\theta = \theta_0 + \omega t + \frac{1}{2}\alpha t^2$ in terms of ω
- To isolate ω , we'll need to get rid of a lot. There's a lot of pieces we need to get rid of, so let's just start getting rid of whatever seems easiest. As always, we'll basically do PEMDAS in reverse.
- $\theta - \theta_0 = \omega t + \frac{1}{2}\alpha t^2$
- Start by subtracting θ_0 from both sides.
- $\theta - \theta_0 - \frac{1}{2}\alpha t^2 = \omega t$
- Now subtract the $\frac{1}{2}\alpha t^2$. We almost have ω by itself, it's just being multiplied by t .
- $\frac{\theta - \theta_0 - \frac{1}{2}\alpha t^2}{t} = \omega$
- Divide both sides by t .
- $\omega = \frac{\theta - \theta_0}{t} - \frac{1}{2}\alpha t$
- Separate the fraction on the right, to reduce $\frac{1}{2}\alpha \frac{t^2}{t} = \frac{1}{2}\alpha t$

1.3 Chapter 4 Linear Equations Answers

Explanations for Chapter 4 Practice Drill 1 - Slope

-
1. $m = 1$ Since the equation $y = x + 5$ is in $y = mx + b$ form, the slope is whatever is multiplying the x : in this case, the coefficient is that invisible 1: $y = x + 5$ is the same as $y = 1x + 5$.
-
2. $m = \frac{2}{7}$ The equation is already in $y = mx + b$ form, so the slope is what's multiplying the x term, $\frac{2}{7}$.
-
3. $\frac{1 - (-1)}{3 - 0} = \frac{2}{3}$ Use the points marked on the graph. I'm using $(0, -1)$ and $(3, 1)$, but you could also use the point $(-3, 3)$ if you'd like. You'll get the same slope with any 2 of the points. Then use the slope formula $\frac{y_2 - y_1}{x_2 - x_1}$, and the slope is $\frac{2}{3}$.
-
4. $\frac{2 - 3}{2 - 0} = \frac{-1}{2}$ Use the points marked on the graph. I'm using $(0, 3)$ and $(2, 2)$, but you could also use the point $(-2, 4)$ if you'd like. You'll get the same slope with any 2 of the points. Then use the slope formula $\frac{y_2 - y_1}{x_2 - x_1}$, and the slope is $-\frac{1}{2}$.
-
5. $\frac{14 - 12}{8 - 5} = \frac{2}{3}$ Use the points given with the slope formula $\frac{y_2 - y_1}{x_2 - x_1}$, and the slope is $\frac{2}{3}$.
-
6. $(6, 11)$ and $(10, 3)$ Although this is in function form (which you can read more about in Chapter 12: Functions and Polynomials on page 1), we're still just given two points. Remember that $f(x) = y$, so anything inside the parentheses is our x -coordinate and anything outside is the y -coordinate. So $f(6) = 11$ is the point $(6, 11)$ and $f(10) = 3$ is the point $(10, 3)$.
- $\frac{3 - 11}{10 - 6} = \frac{-8}{4} = -2$ Use those points with the slope formula $\frac{y_2 - y_1}{x_2 - x_1}$, and the slope is -2 .
-
7. $2y = 9x + 1$
 $y = \frac{9}{2}x + \frac{1}{2}$ Rearrange this into $y = mx + b$ form by isolating y .
Divide both sides by 2, and the slope is $\frac{9}{2}$.
-
8. $8x - 5y = 40$
 $-5y = -8x + 40$
 $y = \frac{8}{5}x - 8$ Let's rearrange this into $y = mx + b$ form by isolating y .
Subtract $8x$ from both sides.
Divide both sides by -5 , and the slope is $\frac{8}{5}$.
-
9. $y = 3$ This line is already in $y = mx + b$ form, we just don't have a m term. How come? It must mean that $m = 0$. You may have also realized that this is a horizontal line that crosses through the y -axis at 3, and all horizontal lines have a slope of 0.
-

- (0, 0) and (c, d)
- We're given two points. Since the line goes through the origin, it passes through point (0,0). We also know that it goes through point (c, d), but the question only tells us the ratio of $c : d$, but it doesn't actually tell us c or d .
10. $c = 6$ and $d = 4$
- Let's make up a value for c . I'm using 6 because it works well with the ratio. If $c = 6$ then $d = 4$. Now we have the point (6, 4).
- $\frac{4-0}{6-0} = \frac{4}{6} = \frac{2}{3}$
- Use the points given with the slope formula $\frac{y_2 - y_1}{x_2 - x_1}$, and the slope is $\frac{2}{3}$.

Explanations for Chapter 4 Practice Drill 2 - Writing Equations

1. $m = -6$
- We're given the slope, so we can start writing our $y = mx + b$ equation, with $m = -6$, and $y = -6x + b$.
- $b = 8$
- The point (0, 8) is, conveniently enough, the y-intercept, so we know the full equation: $y = -6x + 8$.
-
2. $m = 1$
- We're given the slope, so we can start writing our $y = mx + b$ equation, with $m = 1$, and $y = x + b$.
- $b = -3$
- The point (0, -3) is the y-intercept so we know the full equation: $y = x - 3$.
-
3. $m = 2$
- We're given the slope, so we can start writing our $y = mx + b$ equation, with $m = 2$, and $y = 2x + b$.
- $7 = 2(-3) + b$
- Plug in point (-3, 7) into the equation we have so far.
- $7 = -6 + b$
- $13 = b$
- Add 6 to both sides to find b , and the equation is $y = 2x + 13$.
-
4. $\frac{-1-5}{0-(-2)} = \frac{-6}{2} = -3$
- Use the points given with the slope formula $\frac{y_2 - y_1}{x_2 - x_1}$, and the slope is -3 . Now we can start writing our $y = mx + b$ equation, with $m = -3$, and $y = -3x + b$.
- $b = -1$
- We can use either point to find b , but since (0, -1) is the y-intercept, that gives us $b = -1$ directly, and the equation is $y = -3x - 1$.
-
5. $m = \frac{4}{7}$
- We're given the slope, so we can start writing our $y = mx + b$ equation, with $m = \frac{4}{7}$, so $y = \frac{4}{7}x + b$.
- $b = 6$
- We're also given the y-intercept, at (0, 6), so the equation is $y = \frac{4}{7}x + 6$.
-
6. $(-3, 2)$ and $(0, 12)$
- Although this is in function form (which you can read more about in see Chapter 12: Functions and Polynomials on page 1), we're still just given two points. Remember that $f(x) = y$, so anything inside the parentheses is our x -coordinate and anything outside is the y -coordinate. So $f(-3) = 2$ is the point (-3, 2) and $f(0) = 12$ is the point (0, 12).
- $\frac{12-2}{0-(-3)} = \frac{10}{3}$
- Use the points given with the slope formula $\frac{y_2 - y_1}{x_2 - x_1}$, and the slope is $\frac{10}{3}$. Now we can start writing our $y = mx + b$ equation, with $m = \frac{10}{3}$, and $y = \frac{10}{3}x + b$.
- $b = 12$
- We're given the y-intercept at (0, 12), so we can write the equation, $y = \frac{10}{3}x + 12$.

7. $\frac{8-7}{15-12} = \frac{1}{3}$
- Use the points given with the slope formula $\frac{y_2 - y_1}{x_2 - x_1}$, and the slope is $\frac{1}{3}$. Now we can start writing our $y = mx + b$ equation, with $m = \frac{1}{3}$, and $y = \frac{1}{3}x + b$.
- $7 = \frac{1}{3}(12) + b$
- To solve for b , use either point. I used (12, 7) because it's the first one I saw; it really doesn't matter which one you pick. Use whatever point seems easiest.
- $7 = 4 + b$
- $3 = b$
- Subtract 4 from both sides, and the equation is $y = \frac{1}{3}x + 3$.

8. $\frac{0-24}{7-(-1)} = \frac{-24}{8} = -3$
- Use the points given with the slope formula $\frac{y_2 - y_1}{x_2 - x_1}$, and the slope is -3 . Now we can start writing our $y = mx + b$ equation, with $m = -3$, and $y = -3x + b$.
- $0 = -3(7) + b$
- Now use either point to find the y-intercept. Unfortunately, the point (7,0) is the x-intercept, so it doesn't directly tell us b . But, it's still an easy point to plug into our equation.
- $0 = -21 + b$
- $21 = b$
- Add 21 to both sides, and the equation is $y = -3x + 21$.

9. $\frac{156-120}{7-6} = \frac{36}{1} = 36$
- Use the points given with the slope formula $\frac{y_2 - y_1}{x_2 - x_1}$, and the slope is 36. Now we can start writing our $y = mx + b$ equation, with $m = 36$, and $y = 36x + b$.
- $120 = 36(6) + b$
- Use either point to find b . I'm using the first point (6, 120).
- $120 = 216 + b$
- $-96 = b$
- Subtract 216 from both sides, and the equation is $y = 36x - 96$.

10. $m = 0$
- The slope is 0, so this is a horizontal line with an equation $y = 0x + b$, which is the same as $y = b$.
- $8 = b$
- Plug in the point (7, 8) to find b . There's only a spot to put the y-coordinate, so $b = 8$ and the equation is $y = 8$.

Explanations for Chapter 4 Practice Drill 3 - Parallel and Perpendicular Lines

1. $m = -\frac{1}{2}$
- Since line ℓ and line k are parallel, they have the same slope, so the slope of line k is $-\frac{1}{2}$ and the equation so far is $y = -\frac{1}{2}x + b$.
- $4 = -\frac{1}{2}(3) + b$
- Plug in point (3, 4).
- $4 = -\frac{3}{2} + b$
- $\frac{11}{2} = b$
- Add $\frac{3}{2}$ to both sides, and the equation is $y = -\frac{1}{2}x + \frac{11}{2}$.

2. $\frac{5-0}{2-0} = \frac{5}{2}$
- We're given two points: (2, 5) and the origin, (0,0). Use the points given with the slope formula $\frac{y_2 - y_1}{x_2 - x_1}$, and the slope of ℓ is $\frac{5}{2}$. Since line k is perpendicular to line ℓ , they have negative reciprocal slopes, so the slope of line k is $-\frac{2}{5}$. Now we can start writing our $y = mx + b$ equation for line k : $y = -\frac{2}{5}x + b$.
- $b = 10$
- We know the y-intercept for line k , (0, 10), and so the equation is $y = -\frac{2}{5}x + 10$.

-
3. $m = \frac{3}{7}$
- We know the slope of line ℓ is $\frac{3}{7}$ since we're given the $y = mx + b$ equation. Since lines ℓ and k are parallel, they have the same slope, so we can start writing the equation for line k : $y = \frac{3}{7}x + b$.
3. $19 = \frac{3}{7}(-14) + b$
- Use the point $(-14, 19)$ with $y = \frac{3}{7}x + b$.
- $19 = -6 + b$
- $25 = b$
- Add -6 to both sides, and the equation is $y = \frac{3}{7}x + 25$.
-
4. $5x + 2y = 3$
- Since lines ℓ and k are parallel, they have the same slope. Let's use the equation for line ℓ to find the slope of both of those lines.
- $2y = -5x + 3$
- We want to isolate y , so start by subtracting $5x$ from both sides.
- $y = -\frac{5}{2}x + \frac{3}{2}$
- Divide both sides by 2. The slope of line ℓ is $-\frac{5}{2}$, and since lines ℓ and k are parallel, both lines have a slope of $-\frac{5}{2}$, so the equation for k will be $y = -\frac{5}{2}x + b$.
4. $1 = -\frac{5}{2}(2) + b$
- Find b by plugging in the only point we know on line k , $(2, 1)$.
- $1 = -5 + b$
- $6 = b$
- Add 5 to both sides, and $b = 6$. The equation is therefore $y = -\frac{5}{2}x + 6$.
-
5. $6x + 2y = 10$
- Lines ℓ and k are perpendicular, so if we find the slope of ℓ we know the slope of k will be the negative reciprocal. Let's rearrange the equation for ℓ into $y = mx + b$ form.
- $2y = -6x + 10$
- Subtract $6x$ from both sides.
- $y = -3x + 5$
- Divide both sides by 2. Now we know the slope of line ℓ is -3 , which means the slope of the perpendicular line k is $\frac{1}{3}$, and the equation is $y = \frac{1}{3}x + b$.
- $(2, a)$
- To find b , we're going to need a point on line k . Unfortunately, the only point they give us is $(2, a)$, which is only sort of half a point, since we don't know the y -coordinate. However, we know that $(2, a)$ is a point on *both* lines, since it's the point of intersection.
5. $a = -3(2) + 5$
- Plug $(2, a)$ into the equation for line ℓ , $y = -3x + 5$. (You could also use the original, un-rearranged equation. Up to you.)
- $a = -6 + 5$
- $a = -1$
- Now we know the point of intersection is $(2, -1)$.
- $-1 = \frac{1}{3}(2) + b$
- Plug the point $(2, -1)$ into the equation we have so far for k , $y = \frac{1}{3}x + b$.
- $-1 = \frac{2}{3} + b$
- $-\frac{5}{3} = b$
- Subtract $\frac{2}{3}$ from both sides, and the equation is $y = \frac{1}{3}x - \frac{5}{3}$.

Explanations for Chapter 4 Practice Drill 4 - Identify the Graph

-
1. $b = -1$
- The equation is already in $y = mx + b$ form, so we can find the slope and y -intercept easily. Since the y -intercept is -1 , eliminate all the answers that don't cross that y -axis at -1 , and we're left with (A) and (F).
1. $m = 2$
- The slope is positive 2, which means we should go over 1 and up 2. Answer choice (A) has a slope of 1, so the answer is (F), which has a slope of positive 2.

-
2. $b = 2$
 $m = -2$
- The equation is already in $y = mx + b$ form, so we can find the slope and y -intercept easily. Since the y -intercept is 2, eliminate all the answers that don't cross that y -axis at 2, leaving (B), (D), (G), (H), (I), and (K).
- Since the slope is negative, eliminate any lines with a positive slope, that go up from left to right. (B) and (H) are both gone, leaving (D), (G), (I), and (K). Eliminate (I) and (K) because they have fractional slopes: they go over more than they go down. (If you're not sure, you can could out the slope of each of them, but it's a good idea to be able to quickly recognize when a slope is bigger or smaller than 1, or bigger or smaller than -1 .) (D) has a slope of -1 , so the answer is (G), which has a slope of -2 .
-
3. $b = 2$
 $m = -1$
- The equation is already in $y = mx + b$ form, so we can find the slope and y -intercept easily. Since the y -intercept is 2, eliminate all the answers that don't cross that y -axis at 2, leaving (B), (D), (G), (H), (I), and (K).
- Notice that our m term is -1 , since the equation is $y = -x + 2$. Since the slope is negative, eliminate any lines with a positive slope, that go up from left to right. (B) and (H) are both gone, leaving (D), (G), (I), and (K). Eliminate (I) and (K) because they have fractional slopes: they go over more than they go down. (If you're not sure, you can could out the slope of each of them, but it's a good idea to be able to quickly recognize when a slope is bigger or smaller than 1, or bigger or smaller than -1 .) (D) has a slope of -1 , and that's our answer.
-
4. $b = -2$
 $m = -1$
- The equation is already in $y = mx + b$ form, so we can find the slope and y -intercept easily. Since the y -intercept is -2 , eliminate all the answers that don't cross that y -axis at -2 , leaving (C), (E), (J), and (L).
- Notice that our m term is -1 , since the equation is $y = -x - 2$. Since the slope is negative, eliminate any lines with a positive slope, that go up from left to right. (C), (J), and (L) are out, leaving only (E).
-
5. $b = -1$
 $m = 1$
- The equation is already in $y = mx + b$ form, so we can find the slope and y -intercept easily. Since the y -intercept is -1 , eliminate all the answers that don't cross that y -axis at -1 , and we're left with (A) and (F).
- Notice that our m term is 1, since the equation is $y = x - 1$, and we just have a single x term. The slope is positive, but both (A) and (F) are positive, so that doesn't help us too much. Check to see which has a slope of 1: (A) has a slope of 1, so that's our answer.
-
6. $b = 2$
 $m = 1$
- The equation is already in $y = mx + b$ form, so we can find the slope and y -intercept easily. Since the y -intercept is 2, eliminate all the answers that don't cross that y -axis at 2, leaving (B), (D), (G), (H), (I), and (K).
- Notice that our m term is 1, since the equation is $y = x + 2$, and we just have a single x term. Eliminate any negative slopes: (D), (G), (I), and (K) are gone, leaving only (B) or (H) as answers. We need a slope of positive 1, which is answer choice (B).
-

- $b = 2$
- The equation is already in $y = mx + b$ form, so we can find the slope and y -intercept easily. Since the y -intercept is 2, eliminate all the answers that don't cross that y -axis at 2, leaving (B), (D), (G), (H), (I), and (K).
7. $m = -\frac{1}{2}$
- Notice that our m term is $-\frac{1}{2}$, since the equation is $y = -\frac{1}{2}x + 2$. Since the slope is negative, eliminate any lines with a positive slope, that go up from left to right. (B) and (H) are both gone, leaving (D), (G), (I), and (K). Only (I) and (K) have fractional slopes. (I) goes over 2 every time it goes down 1, for a slope of $-\frac{1}{2}$, and (I) is our answer.

- $b = 2$
- The equation is already in $y = mx + b$ form, so we can find the slope and y -intercept easily. Since the y -intercept is 2, eliminate all the answers that don't cross that y -axis at 2, leaving (B), (D), (G), (H), (I), and (K).
8. $m = -\frac{1}{4}$
- Notice that our m term is $-\frac{1}{4}$, since the equation is $y = -\frac{1}{4}x + 2$. Since the slope is negative, eliminate any lines with a positive slope, that go up from left to right. (B) and (H) are both gone, leaving (D), (G), (I), and (K). Only (I) and (K) have fractional slopes. (K) goes over 4 every time it goes down 1, for a slope of $-\frac{1}{4}$, so (K) is the answer.

- $x - 4y = 8$
- Start by rearranging this into $y = mx + b$ form.
- $-4y = -x + 8$
- Subtract x from both sides.
- $y = \frac{1}{4}x - 2$
- Divide both sides by -4 to isolate y .
9. $b = -2$
- The equation is now in $y = mx + b$ form, so we can find the slope and y -intercept easily. Since the y -intercept is -2 , eliminate all the answers that don't cross that y -axis at -2 , leaving (C), (E), (J), and (L).
- $m = \frac{1}{4}$
- The slope is positive, so eliminate any negative slopes: (E) is gone, leaving (C), (J) and (L). (C) has a slope of positive 1, so eliminate that. (L) goes over 4 every time it goes up by 1, so that's a slope of $\frac{1}{4}$, and (L) is our answer.

- $2x - 4y = 8$
- Start by rearranging this into $y = mx + b$ form.
- $-4y = -2x + 8$
- Subtract $2x$ from both sides.
- $y = \frac{1}{2}x - 2$
- Divide both sides by -4 to isolate y .
10. $b = -2$
- The equation is now in $y = mx + b$ form, so we can find the slope and y -intercept easily. Since the y -intercept is -2 , eliminate all the answers that don't cross that y -axis at -2 , leaving (C), (E), (J), and (L).
- $m = \frac{1}{2}$
- The slope is positive, so eliminate (E), leaving (C), (J), and (L). (C) has a slope of positive 1, so eliminate that. (J) goes over 2 every time it goes up by 1, so that's a slope of $\frac{1}{2}$, and (J) is our answer.

Explanations for Chapter 4 Practice Drill 5 - Meaning in Context

- $m = 0.76$
- Since the equation $h(t) = 0.76t + 8.34$ is already in $y = mx + b$ form, 0.76 is the slope of the line. The slope is always a rate, an increase or decrease as things change. Eliminate (A), (B), (C), and (G), leaving (D), (E) and (F). Our y -coordinate is the height of the plant and our x coordinate is time, so the slope, the change in y over the change in x , is the change in height for each change in time, answer choice (F).
- 1.

-
2. $b = 8.34$ Since the equation $h(t) = 0.76t + 8.34$ is already in $y = mx + b$ form, 8.34 is the y -intercept of the line. The y -intercept is always our starting point, which is answer choice (A). To be more specific, it's the height (the y -coordinate) when $t = 0$ (when our x -coordinate is zero, on the y -axis).
-
3. $t = 25$ For $h(25)$, we've replaced the t in $h(t)$ with 25, so $t = 25$. Our function $h(t)$ will therefore tell us the height on day 25, answer choice (B).
-
4. $h(50) - h(0)$ Let's figure out what each part of this means. $h(0)$ is the height of the plant when $t = 0$, or our starting height. (This is the same as the y -intercept.) $h(50)$ is the height after 50 days. So $h(50) - h(0)$ is the difference between the height after 50 days and the height after 0 days, which is the total amount that the plant has grown, answer choice (E).
-
5. $b = 954$ The equation is, with a bit of rearranging, in $y = mx + b$ form: $d = -480t + 954$. That means that 954 is our y -intercept. Since our y -coordinate is the distance, in miles, from Los Angeles, our y -intercept is the distance from Los Angeles when $t = 0$, when we've just started flying from Seattle, answer choice (A).
-
6. $m = -480$ The equation is, with a bit of rearranging, in $y = mx + b$ form: $d = -480t + 954$. So our slope is -480 . With questions about distance and time, our slope is always the velocity (or speed), because slope is $\frac{\text{change in } y}{\text{change in } x} = \frac{\text{distance traveled}}{\text{time}} = \text{speed}$. So 480 is the speed the plane is flying at, so it's either (C) or (D). Our units for distance were miles, and our units for time were hours, so the speed is in miles per hour, answer choice (D).
-
7. $d = 0$ To find out the t -intercept, first find out if t is our x - or y -coordinate. Since t is time, it's our x -coordinate, and the t -intercept is the same as the x -intercept, the point where $y = 0$ and we cross the x -axis. The t -intercept is therefore the point where $d = 0$. If $d = 0$, we are 0 miles away from Los Angeles, and the t -intercept is the time it takes to get to Los Angeles, answer choice (F) or (G). Since t is in terms of hours, the answer is (G).
-
8. $y = -12x + 2000$ The setup for the problem gives information about the price required for 1,520 customers. They've already put in $y = 1520$ into the equation, but let's just think of the equation as a $y = mx + b$ equation. x is the price and y is the number of customers.
- $b = 2000$ Our y -intercept is 2000, which means that's the number of customers when our price (x) is zero, answer choice (A).
-
9. $y = -12x + 2000$ The setup for the problem gives information about the price required for 1,520 customers. They've already put in $y = 1520$ into the equation, but let's just think of the equation as a $y = mx + b$ equation. x is the price and y is the number of customers.
- $m = -12$ Our slope is -12 . Slope is always a rate, so the answer is (D), (E), (F), or (G). Since it's negative, it has to be a decrease, so (E) or (G). Let's look at the units. y is customers and x is price, so our slope is $\frac{\text{change in } y}{\text{change in } x} = \frac{\text{change in customers}}{\text{change in price}}$. The change in number customers for every change in price of a dollar, answer choice (G).

1.4 Chapter 5 Systems of Equations Answers

Explanations for Chapter 5 Practice Drill 1 - Substitution

	$7x + 3(5) = 1$	Substitute $y = 5$ into the first equation.
	$7x + 15 = 1$	
1.	$7x = -14$	Subtract 15 from both sides.
	$x = -2$	Divide both sides by 7. Since the second equation already gave us $y = 5$, we don't have to solve for it, and the answer is $(-2, 5)$.

	$x + 4 = 10$	The first equation only has one variable, so let's use it to isolate x .
	$x = 6$	Subtract 4 from both sides.
2.	$y - 8(6) = 10$	Substitute $x = 6$ into the second equation.
	$y - 48 = 10$	
	$y = 58$	Add 48 to both sides. $(6, 58)$.

	$3y = 15$	The first equation only has one variable, so let's use it to isolate y .
	$y = 5$	Divide both sides by 3.
3.	$4x - 5(5) = 7$	Substitute in $y = 5$ to the second equation.
	$4x - 25 = 7$	
	$4x = 32$	Add 25 to both sides.
	$x = 8$	Divide both sides by 4, and the answer is $(8, 5)$.

	$\frac{x}{3} = 2$	The second equation only has one variable, so let's use it to isolate x .
	$x = 6$	Multiply both sides by 3.
4.	$2y + 3(6) = 0$	Substitute in $x = 6$ to the first equation.
	$2y + 18 = 0$	
	$2y = -18$	Subtract 18 from both sides.
	$y = -9$	Divide both sides by 2, and the answer is $(6, -9)$.

	$y - 5x = -17$	The second equation almost has y by itself, so let's use it and isolate y .
	$y = 5x - 17$	Add $5x$ to both sides.
	$2(5x - 17) = 4x - 10$	Substitute in $y = 5x - 17$ to the second equation.
	$10x - 34 = 4x - 10$	Distribute the 2.
5.	$10x = 4x + 24$	Add 34 to both sides.
	$6x = 24$	Subtract $4x$ from both sides.
	$x = 4$	Divide both sides by 6.
	$y - 5(4) = -17$	Substitute in $x = 4$ to the second equation.
	$y - 20 = -17$	
	$y = 3$	Add 20 to both sides, and the answer is $(4, 3)$

	$x - 3y = 23$	Use either equation to isolate for whatever variable seems easiest. Either isolating y in the first equation or x in the second equation seems easy, since neither is multiplied by anything. I'm using the second equation to isolate for x .
	$x = 3y + 23$	Add $3y$ to both sides.
	$y + 26 = 2(3y + 23)$	Substitute in $x = 3y + 23$ to the first equation.
	$y + 26 = 6y + 46$	Distribute.
6.	$26 = 5y + 46$	Subtract y from both sides.
	$-20 = 5y$	Subtract 46 from both sides.
	$-4 = y$	Divide both sides by 5.
	$-4 + 26 = 2x$	Substitute in $y = -4$ to either equation to solve for x .
	$22 = 2x$	
	$11 = x$	Divide both sides by 2, and the answer is $(11, -4)$.

	$2y = 6x + 12$	The first equation is nearly already isolated for y , so let's use that.
	$y = 3x + 6$	Divide both sides by 2.
	$5x + 5(3x + 6) = 10$	Substitute $y = 3x + 6$ into the second equation.
	$5x + 15x + 30 = 10$	Distribute the 5.
	$20x + 30 = 10$	
7.	$20x = -20$	Subtract 30 from both sides.
	$x = -1$	Divide both sides by 20.
	$2y = 6(-1) + 12$	Substitute in $x = -1$ to the first equation.
	$2y = -6 + 12$	
	$2y = 6$	
	$y = 3$	Divide both sides by 2, and the answer is $(-1, 3)$.

- 8.
- | | |
|--------------------------------|--|
| $y + 2 = \frac{x}{3}$ | The first equation is nearly already isolated for y , so let's use that. |
| $y = \frac{x}{3} - 2$ | Subtract 2 from both sides. |
| $2(\frac{x}{3} - 2) + 26 = 8x$ | Substitute in $y = \frac{x}{3} - 2$ to the second equation. |
| $\frac{2x}{3} - 4 + 26 = 8x$ | Distribute the 2. |
| $\frac{2x}{3} + 22 = 8x$ | |
| $2x + 66 = 24x$ | Multiply the entire equation by 3 to get rid of the fraction. |
| $66 = 22x$ | Subtract $2x$ from both sides. |
| $3 = x$ | Divide both sides by 22. |
| $y + 2 = \frac{3}{3}$ | Substitute in $x = 3$ to the first equation. |
| $y + 2 = 1$ | |
| $y = -1$ | Subtract 2 from both sides, and the answer is $(3, -1)$. |

-
- 9.
- | | |
|---------------------------|---|
| $2y = -6x + 34$ | The second equation is nearly already isolated for y , so let's use that. |
| $y = -3x + 17$ | Divide both sides by 2. |
| $15((-3x + 17) - 1) = 3x$ | Substitute in $y = -3x + 17$ to the first equation. |
| $15(-3x + 16) = 3x$ | |
| $-45x + 240 = 3x$ | Distribute the 15. |
| $240 = 48x$ | Add $45x$ to both sides. |
| $5 = x$ | Divide both sides by 48. |
| $2y = -6(5) + 34$ | Substitute in $x = 5$ to the second equation. |
| $2y = -30 + 34$ | |
| $2y = 4$ | |
| $y = 2$ | Divide both sides by 2, and the answer is $(5, 2)$. |
-

- $6y = 30x + 19$
 $y = 5x + \frac{19}{6}$
 $6x + 3\left(5x + \frac{19}{6}\right) = -1$
 $6x + 15x + \frac{19}{2} = -1$
 $21x + \frac{19}{2} = -1$
 $42x + 19 = -2$
 $42x = -21$
 $x = -\frac{1}{2}$
 $6y = 30\left(-\frac{1}{2}\right) + 19$
 $6y = -15 + 19$
 $6y = 4$
 $y = \frac{4}{6} = \frac{2}{3}$
- Neither of these equations are great, so I'm using the second one because it's closest to already being isolated for y .
 Divide both sides by 6.
 Substitute in $y = 5x + \frac{19}{6}$ to the first equation.
 Distribute the 3. $3 \times \frac{19}{6} = \frac{19(3)}{6} = \frac{19}{2}$. Remember to always look to simplify fractions before multiplying.
 Multiply both sides by 2 to get rid of the fraction.
 Subtract 19 from both sides.
 Substitute in $x = -\frac{1}{2}$ to the second equation.
 Divide both sides by 6 and reduce. The answer is $\left(-\frac{1}{2}, \frac{2}{3}\right)$.

Explanations for Chapter 5 Practice Drill 2 - Suspicious Systems and Addition

- $3x + 4y = 3$
 $2x + y = 1$
 $5x + 5y = 4$
- The question asks for $5x + 5y$. If we add the equations together, we get exactly that, so $5x + 5y = 4$.
-
- $x + 3y = 14$
 $x - 3y = -6$
 $2x = 8$
 $x = 4$
- The question asks for the value of x . If we add the two equations together we get $2x = 8$, which is pretty close.
 Divide both sides by 2.
-
- $5x - 2y = 11$
 $x - 4y = 19$
 $6x - 6y = 30$
 $x - y = 5$
- The question wants $x - y$. By adding the equations together, we got pretty close.
 Divide both sides by 6.

- $x + 3y = 4$
 $2x + y = 3$

 $3x + 4y = 7$
 $x = -3y + 4$
4. $2(-3y + 4) + y = 3$
 $-6y + 8 + y = 3$
 $-5y + 8 = 3$
 $-5y = -5$
 $y = 1$
 $x + 3(1) = 4$
 $x + 3 = 4$
 $x = 1$
 $x + y = 1 + 1 = 2$
- We want $x + y$, but adding up the equations doesn't give us anything close to that.
- Back to substitution then. I'm using the first equation and isolating x by subtracting $3y$ from both sides.
- Substitute in $x = -3y + 4$ to the second equation.
- Distribute the 2.
- Subtract 8 from both sides.
- Divide both sides by -5 .
- Substitute in $y = 1$ to the first equation.
- Subtract 3 from both sides.
- Now that we know x and y , find $x + y$.
-

5. $3x - y = 21$
 $x + 5y = 23$

 $4x + 4y = 44$
 $x + y = 11$
- The question wants to know the value of $x + y$. Adding the two equations gets us pretty close to that.
- Divide both sides by 4.
-

6. $8x + y = 5$
 $2x + 3y = 1$

 $10x + 4y = 6$
 $50x + 20y = 30$
- The question wants to know the value of $50x + 20y$. We didn't get that, but we did get $\frac{1}{5}$ of that.
- Multiply both sides by 5.
-

7. $3x + 5y = 7$
 $x + y = 9$

 $4x + 6y = 16$
 $2x + 3y = 8$
- The question wants to know the value of $2x + 3y$. By adding the equations, we got double that.
- Divide both sides by 2.
-

$$6x - y = 5$$

$$\frac{4x - 4y = 8}{10x - 5y = 13}$$

$$10x - 5y = 13$$

$$-y = -6x + 5$$

$$y = 6x - 5$$

$$4x - 4(6x - 5) = 8$$

$$4x - 24x + 20 = 8$$

$$-20x + 20 = 8$$

8. $-20x = -12$

$$x = \frac{12}{20} = \frac{3}{5}$$

$$6\left(\frac{3}{5}\right) - y = 5$$

$$\frac{18}{5} - y = 5$$

$$18 - 5y = 25$$

$$-5y = 7$$

$$y = -\frac{7}{5}$$

$$5x + 5y = 5\left(\frac{3}{5}\right) + 5\left(-\frac{7}{5}\right)$$

$$3 + -7 = -4$$

The question wants to know the value of $5x + 5y$. Adding the equations together doesn't give us anything that looks like that, so we're going to have to use substitution.

Using the first equation, we can start to isolate for y by subtracting $6x$ from both sides.

Multiply both sides by -1 .

Substitute $y = 6x - 5$ into the second equation.

Distribute the -4 .

Subtract 20 from both sides.

Divide both sides by -20 and simplify.

Substitute in $x = \frac{3}{5}$ to the first equation.

Multiply both sides by 5 to get rid of the fraction.

Subtract 18 from both sides.

Divide both sides by -5 .

Substitute in the values of x and y that we found.

$$5x - \frac{1}{3}y = 4$$

9. $\frac{x - \frac{1}{3}y = 7}{6x = 11}$

$$6x = 11$$

$$x = \frac{11}{6}$$

The question wants to know the value of x , and adding the equations gives us a single equation in terms of x , so let's use that.

Divide both sides by 6.

- 10.
- $$\frac{1}{3}x + y = 1$$
- $$x + \frac{1}{2}y = 3$$
-
- $$\frac{4}{3}x + \frac{3}{2}y = 4$$
- $$x = -\frac{1}{2}y + 3$$
- The question asks for $4x + 3y$, and adding the equations we have doesn't give us that. It looks sort of like $4x + 3y$, but we can't get it in exactly that form from here. Sigh. Looks like we have to do substitution.
- Isolate one of the variables. I'm isolating x in the second equation, but you could also isolate y in the first equation just as easily.
- $$\frac{1}{3}\left(-\frac{1}{2}y + 3\right) + y = 1$$
- Substitute in $x = -\frac{1}{2}y + 3$ to the first equation.
- $$-\frac{1}{6}y + 1 + y = 1$$
- Distribute the $\frac{1}{3}$
- $$\frac{5}{6}y + 1 = 1$$
- Subtract 1 from both sides.
- $$\frac{5}{6}y = 0$$
- Multiply both sides by $\frac{6}{5}$.
- $$y = 0$$
- Substitute in $y = 0$ to the second equation.
- $$x + \frac{1}{2}(0) = 3$$
- $$x + 0 = 3$$
- $$x = 3$$
- Use the values for x and y to find $4x + 3y$.
- $$4x + 3y = 4(3) + 3(0) = 12 + 0 = 12$$

Explanations for Chapter 5 Practice Drill 3 - Write But Don't Solve

- 1.
- $$S + L = 8$$
- The quantity equation: there are 8 total sodas, so the small sodas S plus the large sodas L equals 8.
- $$2S + 3L = 21$$
- Each small soda is \$2, so the total cost for small sodas is $2S$. Each large soda is \$3, so the total cost for large sodas is $3L$. The total amount spent is $2S + 3L = 21$.
-
- 2.
- $$b + f = 40$$
- The quantity equation: the shop sold 40 total posters, so the black-and-white posters b plus the full-color posters, f , equals 40.
- $$15b + 25f = 730$$
- The price equation. The total price of all black-and-white posters sold is $15b$, and the total price of all full-color posters sold is $25f$. The total amount sold is therefore $15b + 25f = 730$.
-
- 3.
- $$A + C = 1,249$$
- The quantity equation: the adults plus the children combined is 1,249 total tickets.
- $$80A + 55C = 80,320$$
- The price equation. Each adult ticket is \$80, so the total amount from adult tickets is $80A$, and each child's ticket is \$55, so the total amount from children's tickets is $55C$. Combined, adult tickets and children's tickets made a total of \$80,320.
-
- 4.
- $$n + c = 5$$
- The quantity equation. There are n cups of nuts, plus c cups of candy, which adds up to 5 total cups of snack mix.
- $$600n + 900c = 3900$$
- The total calories equation. This is basically the same as our normal price equation: each cup of nuts has 600 calories, so the total calories from nuts is $600n$, and each cup of candy has 900 calories, so the total calories from candy is $900c$. Combined, they add up to 3900 calories.

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5. $t + b = 56$
 $3t + 7b = 322$
- The quantity equation. The tacos plus the burritos add up to 56.
- The price equation. Each taco is \$3, so tacos made a total of $3t$ dollars, and each burrito is \$7, so burritos made a total of $7b$ dollars. The taco truck made a total of \$322 in an hour, so the total amount from tacos plus the total amount from burritos adds up to 322.
-
6. $b \leq 4$
 $b + v \geq 5$
 $30b + 8v = 120$
- "Susan hires no more than 4 buses." The number of buses could be less than 4, it could equal 4, but it can't be more.
- The quantity inequality, just like our quantity equations in the earlier problems. In this case, however, the number of vehicles, $b + v$, must be "at least 5."
- The equation for the total number of people in buses and vans. Each bus can hold 30 people, so the total number of people in the buses is $30b$. Each van can hold 8 people, so the total number of people riding in vans is $8v$. Combined, the number of people in buses plus the number of people in vans is 120. (This is just like our price equations from the previous problems, just with people.)
-
7. $H + E = 8$
 $10E + 30H = 140$
- The player answered a total of H hard questions correctly and E easy questions correctly. Combined, the player answered 8 questions correctly.
- The points equation. The total number of points from easy questions is the number of easy questions answered correctly multiplied by 10, for $10E$ points total. Each correctly answered hard question is worth 30 points, for $30H$ total. Combined, the points from the easy questions plus the points from the hard questions equals 140.
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8. $x + y = 100$
 $0.017x + 0.07y = 5.145$
- Start with the quantity equation. The total number of small weights, x , plus larger weights, y , is 100.
- The weight equation. Each small weight is 0.017 ounces, so the total weight of all the smaller weights is $0.017x$. Each larger weight is 0.07 ounces, so the total weight of all the larger weights is $0.07y$. The weight of all the small weights plus the weight of all the larger weights adds up to 5.145 ounces.
-
9. $x \geq 2$
 $y \geq 8$
 $x + y \geq 22$
 $4x + 7y \leq 140$
- "The office will order at least 2 plain hamburgers."
- "...and at least 8 fully-loaded hamburgers."
- The quantity equation. The office will order "at least 22 hamburgers," so the total number of hamburgers ordered must be equal to or greater than 22.
- The price equation. The total cost for x plain hamburgers, at \$4 each, is $4x$, and the total cost for y fully-loaded hamburgers, at \$7 each, is $7y$. The office "spends no more than \$140," so the total price of the plain hamburgers plus the fully-loaded hamburgers could be equal to or less than 140.
-
10. $p + s < 25$
 $175p + 22s > 1000$
- The quantity equation. Nathan's goal was to sell "at least 25 photographs," so $p + s \geq 25$ However, "he did not meet his goal." That means that he must have sold fewer than 25 total photographs. Sorry Nathan. Keep at it. You're good, you've got talent.
- The price equation. The total amount from poster-sized prints, $175p$, and the total amount from standard-sized prints, s , was "over \$1000." So it can't be less than \$1000, it can't equal \$1000, it must be greater than \$1000.

Explanations for Chapter 5 Practice Drill 4 - Write and Solve

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|----|------------------------|--|
| 1. | $S + L = 6$ | Let's say the 10-pound weights, the smaller ones, are S , and the 25-pound weights, the larger ones, are L . There are 6 total weights, so we can write the quantity equation. |
| | $10S + 25L = 90$ | Now we can write an equation for the total amount of weight on the barbell. Each 10-pound weight weighs, well, 10 pounds, so the total weight of all S 10-pound weights is $10S$. For 25-pound weights, the total weight is $25L$. Those, together, add up to 90 pounds. |
| | $S = 6 - L$ | Isolate one of the variables so we can substitute. I'm isolating S by subtracting L from both sides, but you could isolate L just as easily. |
| | $10(6 - L) + 25L = 90$ | Substitute in $S = 6 - L$ to the second equation. |
| | $60 - 10L + 25L = 90$ | Distribute the 10. |
| | $60 + 15L = 90$ | Combine like terms. |
| | $15L = 30$ | Subtract 60 from both sides. |
| | $L = 2$ | Divide both sides by 15. |
| | $S + 2 = 6$ | Substitute in $L = 2$ to the first equation to find S . |
| | $S = 4$ | Subtract 2 from both sides. There are 4 10-pound weights and 2 25-pound weights. |

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|----|--------------------------|--|
| 2. | $M + E = 10$ | First, the quantity equation. The number of matinee tickets, M , plus the number of evening tickets, E , is 10. |
| | $12M + 20E = 176$ | The price equation. Each matinee ticket is \$12, so the total amount spent on matinee tickets is $12M$. Each evening ticket is \$20, so the total amount spent on evening tickets is $20E$. Together, they add up to the total amount spent on tickets, \$176. |
| | $M = 10 - E$ | Isolate one of the variables in the first equation. |
| | $12(10 - E) + 20E = 176$ | Substitute in $M = 10 - E$ to the second equation. |
| | $120 - 12E + 20E = 176$ | Distribute the 12. |
| | $120 + 8E = 176$ | Combine like terms. |
| | $8E = 56$ | Subtract 120 from both sides. |
| | $E = 7$ | Divide both sides by 8. |
| | $M + 7 = 10$ | Substitute in $E = 7$ to the first equation. |
| | $M = 3$ | Subtract 7 from both sides. Andrew bought 3 matinee tickets and 7 evening tickets. |
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	$R + F = 110$	The quantity equation. The number of regular subs, R , plus the number of foot-long subs, F , equals 110.
	$9R + 15F = 1350$	The price equation. Each regular sub is \$9, so the total made from regular subs during the lunch rush is $9R$. Each foot-long sub is \$15, so the total made from foot-long subs is $15F$. In total, they made \$1350.
	$R = 110 - F$	Isolate one of the variables in the first equation. Here, I subtracted F from both sides.
3.	$9(110 - F) + 15F = 1350$	Substitute in $R = 110 - F$ to the second equation.
	$990 - 9F + 15F = 1350$	Distribute the 9.
	$990 + 6F = 1350$	Combine like terms.
	$6F = 360$	Subtract 990 from both sides.
	$F = 60$	Divide both sides by 6.
	$R + 60 = 110$	Substitute in $F = 60$ to the first equation.
	$R = 50$	Subtract 60 from both sides. During the lunch rush, the sandwich shop sold 50 regular subs and 60 foot-long subs.

	$R + S = 6$	The quantity equation. In total, the number of cups of raspberries, R , plus the number of cups of strawberries, S , adds up to 6.
	$65R + 50S = 330$	The calories equation, just like our normal price equation. The total number of calories from raspberries is $65R$, and the total number of calories from strawberries is $50S$. In total, they add up to 330 calories.
	$R = 6 - S$	Isolate one of the variables in the first equation. Here, I've subtracted S from both sides to isolate R .
4.	$65(6 - S) + 50S = 330$	Substitute in $R = 6 - S$ into the second equation.
	$390 - 65S + 50S = 330$	Distribute the 65.
	$390 - 15S = 330$	Combine like terms.
	$-15S = -60$	Subtract 390 from both sides.
	$S = 4$	Divide both sides by -15 .
	$R + 4 = 6$	Substitute in $S = 4$ to the first equation.
	$R = 2$	Subtract 4 from both sides. The smoothies has 2 cups of raspberries and 4 cups of strawberries.

	$V + C = 16$	The quantity equation. The number of violins, V , plus the number of cellos, C , adds up to 16.
	$180V + 340C = 3840$	The price equation. Each violin is \$180, so the school spent a total of $180V$ dollars on violins. Each cello is \$340, so the school spent a total of $340C$ on cellos. Combined, the school spent \$3,840 on cellos and violins.
	$V = 16 - C$	Isolate one of the variables in the first equation. Here, I'm isolating V by subtracting C from both sides.
5.	$180(16 - C) + 340C = 3840$	Substitute in $V = 16 - C$ to the second equation.
	$2880 - 180C + 340C = 3840$	Distribute the 180.
	$2880 + 160C = 3840$	Combine like terms.
	$160C = 960$	Subtract 2880 from both sides.
	$C = 6$	Divide both sides by 160.
	$V + 6 = 16$	Substitute $C = 6$ into the first equation.
	$V = 10$	Subtract 6 from both sides. The school bought 10 violins and 6 cellos.

	$P = D + 10$	Let's have P be the price of the print version, and D be the price of the digital version. "The print version is 10 dollars more than the digital version" can be translated directly into an equation. (Unlike in our other systems, here P and D represent the prices of the print and digital copies, rather than the number of copies sold.)
	$3P + 4D = 142$	If each print copy costs P dollars, then 3 print copies will cost $3P$ dollars. Similarly, if each digital copy costs D dollars, then 4 digital copies will cost $4D$ dollars. Together, 3 print copies and 4 digital copies cost \$142.
6.	$3(D + 10) + 4D = 142$	The first equation was already in terms of P , so we can substitute it into the second equation.
	$3D + 30 + 4D = 142$	Distribute the 3.
	$7D + 30 = 142$	Combine like terms.
	$7D = 112$	Subtract 30 from both sides.
	$D = 16$	Divide both sides by 7.
	$P = 16 + 10$	Substitute in $D = 16$ to the first equation.
	$P = 26$	The price of a print version of the book is \$26.

	$3P + 6G = 9$	Here, we don't have the usual quantity and price equations. Instead, we know that "Erin buys 3 slices of pizza and 6 garlic knots for \$9," so we can write an equation for that, using P as the price of a single slice of pizza and G as the price of a single garlic knot. 3 slices of pizza cost $3P$ and 6 garlic knots cost $6G$.
	$2P + 8G = 8$	We also know that "Anne Marie buys 2 slices of pizza and 8 garlic knots for \$8," so we can write an equation using that information. Now we have 2 equations and 2 variables, so we can solve the system.
	$P + 2G = 3$	Adding the equations together doesn't give us anything helpful ($5P + 14G = 17$), so we'll have to substitute. You can use either equation, but I'm using the first one, from Erin. Divide both sides by 3.
	$P = 3 - 2G$	Subtract $2G$ from both sides to isolate P .
7.	$2(3 - 2G) + 8G = 8$	Substitute in $P = 3 - 2G$ to the second equation.
	$6 - 4G + 8G = 8$	Distribute the 2.
	$6 + 4G = 8$	Combine like terms.
	$4G = 2$	Subtract 6 from both sides.
	$G = 0.5$	Divide both sides by 4. Now we know that each garlic knot is 0.5 dollars, or 50 cents.
	$3P + 6(0.5) = 9$	Substitute in $G = 0.5$ to either equation. I'm using the first one.
	$3P + 3 = 9$	
	$3P = 6$	Subtract 3 from both sides.
	$P = 2$	Divide both sides by 3. Each slice of pizza is 2 dollars and each garlic knot is 0.5 dollars.

	$x, x + 1, x + 2$	For problems with consecutive integers (or any sort of sequence), call the first number x . The next number after x is $x + 1$: for instance, if our first number is 6, the next consecutive integer is $6 + 1 = 7$. The third integer is one larger than that, so it's $x + 2$.
8.	$(x + 1) + (x + 2) = 3(x) - 8$	"The sum of the second and third integers is 8 less than three times the first integer." On the left side of the equation, we have the sum of the second integer ($x + 1$) and the third integer ($x + 2$). We know that <i>is</i> (equals) 8 less than three times the first integer, x .
	$2x + 3 = 3x - 8$	Combine like terms.
	$2x + 11 = 3x$	Add 8 to both sides.
	$11 = x$	Subtract $2x$ from both sides. The integers 11, 12, and 13.

- $C + J = 1$
 We know that we have one cup of fruit punch with 180 milligrams of potassium. Let's say C is the number of cups of cranberry juice and J is the number of cups of orange juice. (I was going to use O for orange juice, but that was too close to the number zero, so I'm using J instead.) Together, the cranberry juice and the orange juice add up to that 1 cup of fruit punch.
- $195C + 170J = 180$
 Now an equation for the potassium in each. Each cup of cranberry juice has 195 mg of potassium, so the cranberry juice adds a total of $195C$ mg of potassium. Each cup of orange juice has 170 mg of potassium, so the orange juice adds $170C$ mg of potassium. Together, they add up to the 180 mg of potassium in 1 cup of the fruit punch.
- $C = 1 - J$
 Isolate one of the variables in the first equation. I've subtracted J from both sides to isolate C .
9. $195(1 - J) + 170J = 180$
 Substitute $C = 1 - J$ into the second equation.
- $195 - 195J + 170J = 180$
 Distribute the 195.
- $195 - 25J = 180$
 Combine like terms.
- $-25J = -15$
 Subtract 195 from both sides.
- $J = \frac{15}{25} = \frac{3}{5}$
 Divide both sides by -25 . Now we know there is $\frac{3}{5}$ cups of orange juice.
- $C + \frac{3}{5} = 1$
 Substitute in $J = \frac{3}{5}$ to the first equation.
- $C = \frac{2}{5}$
 Subtract $\frac{3}{5}$ from both sides. The fruit punch has $\frac{2}{5}$ cups of cranberry juice.
- $\frac{2}{5} = 0.4 = 40\%$, $\frac{3}{5} = 0.6 = 60\%$
 Convert each fraction into a percent, out of 1 cup of fruit punch. The fruit punch is 40% cranberry juice and 60% orange juice.

- $\frac{600}{m} = A$
 Let's say that A is the amount that Mark originally agreed to pay Lou back each month. In that case, the total amount, 600, divided by the number of months, m , will give us A , the amount that Mark will pay back each month.
- $\frac{600}{m-2} = A + 25$
 If Mark has to pay back the loan 2 months earlier, then he only has $m - 2$ months to pay back that \$600. In that case, the amount he pays each month, A , will go up by 25. Now we have a system of equations: two variables and two equations.
- $\frac{600}{m-2} = \frac{600}{m} + 25$
 The first equation is already in terms of A , so substitute in $A = \frac{600}{m}$ to the second equation.
- $\frac{600}{m-2} = \frac{600}{m} + \frac{25m}{m}$
 I'm going to multiply 25 by $\frac{m}{m}$, so I can combine the right side into one single fraction.
10. $\frac{600}{m-2} = \frac{600+25m}{m}$
 Now the fractions on the right have the same denominator, we can combine them. Which is nice because now we have a fraction equals a fraction, and we can cross multiply.
- $600m = (m-2)(600+25m)$
 Cross multiply.
- $600m = 600m + 25m^2 - 1200 - 50m$
 FOIL the right side.
- $0 = 25m^2 - 50m - 1200$
 Since we have a $600m$ term on both sides, let's get rid of that now by subtracting $600m$ from both sides.
- $0 = m^2 - 2m - 48$
 Divide both sides by 25.
- $0 = (m-8)(m+6)$
 Factor the quadratic. We need something that multiplies to -48 and adds to -2 . $-8 \times 6 = -48$ and $-8 + 6 = -2$, so the factors are -8 and 6 .
- $m = 8$ or $m = -6$
 We have two possible values for m . Since Mark probably didn't decide to pay Lou back over -6 months, the answer is $m = 8$.

Explanations for Chapter 5 Practice Drill 5 - Points of Intersection

1.	$2x - 5 = x + 1$ $x - 5 = 1$ $x = 6$ $y = 6 + 1 = 7$	<p>Set the two equations equal to each other to find the point of intersection.</p> <p>Subtract x from both sides.</p> <p>Add 5 to both sides.</p> <p>Substitute in $x = 6$ to either equation to solve for y. I'm using the second equation, since it's a bit simpler. The intersection is at $(6, 7)$.</p>
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2.	$-x + 5 = 3x - 1$ $5 = 4x - 1$ $6 = 4x$ $\frac{6}{4} = 1.5 = x$ $y = -(1.5) + 5$ $y = 3.5$	<p>Set the two equations equal to each other to find the point of intersection.</p> <p>Add x to both sides.</p> <p>Add 1 to both sides.</p> <p>Divide both sides by 4.</p> <p>Substitute in $x = 1.5$ to either equation. The first equation looks a bit simpler, so I'm using that one.</p> <p>The intersection is at $(1.5, 3.5)$.</p>
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3.	$3(-x + 1) = 5x$ $-3x + 3 = 5x$ $3 = 8x$ $\frac{3}{8} = x$ $y = -\frac{3}{8} + 1$ $y = \frac{5}{8}$	<p>Since the second equation is already in terms of y, substitute in $y = -x + 1$ to the first equation.</p> <p>Distribute the 3.</p> <p>Add $3x$ to both sides.</p> <p>Divide both sides by 8.</p> <p>Use either equation to find y. I'm using the second. Substitute in $x = \frac{3}{8}$</p> <p>The intersection is at $(\frac{3}{8}, \frac{5}{8})$.</p>
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4.	$\frac{1}{2}x - 2 = 2x + \frac{1}{3}$ $3x - 12 = 12x + 2$ $-12 = 9x + 2$ $-14 = 9x$ $-\frac{14}{9} = x$ $y = \frac{1}{2}\left(-\frac{14}{9}\right) - 2$ $y = -\frac{7}{9} - 2$ $y = -\frac{25}{9}$	<p>Set the two equations equal to each other to find the point of intersection.</p> <p>Multiply everything by 6 to get rid of the fractions.</p> <p>Subtract $3x$ from both sides.</p> <p>Subtract 2 from both sides.</p> <p>Divide both sides by 9.</p> <p>Substitute in $x = -\frac{14}{9}$ to either equation. I'm using the first one.</p> <p>The point of intersection is $(-\frac{14}{9}, -\frac{25}{9})$.</p>
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- 5.
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|----------------------|--|
| $7x + 2(3x - 5) = 3$ | Since the second equation is already in terms of y , substitute in $y = 3x - 5$ to the first equation to find the point of intersection. |
| $7x + 6x - 10 = 3$ | Distribute the 2. |
| $13x - 10 = 3$ | Combine like terms. |
| $13x = 13$ | Add 10 to both sides. |
| $x = 1$ | Divide both sides by 13. |
| $y = 3(1) - 5$ | Substitute in $x = 1$ to either equation. The second one looks a bit easier to me, so I'm using that one. |
| $y = -2$ | The point of intersection is $(1, -2)$. |
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- 6.
- | | |
|------------------------|---|
| $5(3(y - 1) + 3) = 2y$ | The second equation is already in terms of x , so substitute in $x = 3(y - 1)$ to the first equation. |
| $5(3y - 3 + 3) = 2y$ | Distribute the 3 on the inner parentheses. |
| $15y = 2y$ | Combine like terms. |
| $13y = 0$ | Subtract $2y$ from both sides. $y = 0$. |
| $3(0 - 1) = x$ | Substitute in $y = 0$ to either equation. I'm using the second one. |
| $-3 = x$ | The point of intersection is $(-3, 0)$. |
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- 7.
- | | |
|------------------------|--|
| $2x = 1 - 6y$ | Isolate for one of the variables in either equation. I'm isolating x in the second equation, because that seemed easiest. Subtract $6y$ from both sides. |
| $x = 0.5 - 3y$ | Divide both sides by 2. |
| $3(0.5 - 3y) + 2y = 5$ | Substitute in $x = 0.5 - 3y$ to the first equation. |
| $1.5 - 9y + 2y = 5$ | Distribute the 3. |
| $1.5 - 7y = 5$ | Combine like terms. |
| $-7y = 3.5$ | Subtract 1.5 from both sides. |
| $y = -0.5$ | Divide both sides by -7 . |
| $3x + 2(-0.5) = 5$ | Substitute in $y = -0.5$ to either equation. I'm using the first one. |
| $3x - 1 = 5$ | |
| $3x = 6$ | Add 1 to both sides. |
| $x = 2$ | Divide both sides by 3. The point of intersection is $(2, -0.5)$. |
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8.	$y = -0.75x - 0.4$	Use the first equation to isolate for y by subtracting 0.4 from both sides.
	$5(-0.75x - 0.4) - 1 = 15x + 12$	Substitute in $y = -0.75x - 0.4$ to the second equation.
	$-3.75x - 2 - 1 = 15x + 12$	Distribute the 5.
	$-3.75x - 3 = 15x + 12$	Combine like terms.
	$-3 = 18.75x + 12$	Add $3.75x$ to both sides.
	$-15 = 18.75x$	Subtract 12 from both sides.
	$-0.8 = x$	Divide both sides by 18.75.
	$y + 0.4 = -0.75(-0.8)$	Substitute in $x = -0.8$ to the first equation.
	$y + 0.4 = 0.6$	
	$y = 0.2$	Subtract 0.4 from both sides of the equation. The point of intersection is $(-0.8, 0.2)$.

9.	$-2y = 6 - 5x$	Isolate one of the variables in either equation. Isolating for y in the first equation looked easiest to me, since we only have to divide by -2 .
	$y = -3 + 2.5x$	Speaking of which, divide both sides by -2 .
	$7x - 3(-3 + 2.5x) = 4$	Substitute in $y = -3 + 2.5x$ to the second equation.
	$7x + 9 - 7.5x = 4$	Distribute the -3 .
	$-0.5x + 9 = 4$	Combine like terms.
	$-0.5x = -5$	Subtract 9 from both sides.
	$x = -10$	Divide both sides by -0.5 (which is the same as multiplying both sides by -2).
	$5(10) - 2y = 6$	Substitute in $x = 10$ to either equation.
	$50 - 2y = 6$	
	$-2y = -44$	Subtract 50 from both sides.
$y = 22$	Divide both sides by -2 . The point of intersection is $(10, 22)$.	

10.	$0.2x + 3(0.1x - 2) = 1$	Since the first equation is already in terms of y , substitute $y = 0.1x - 2$ into the second equation.
	$0.2x + 0.3x - 6 = 1$	Distribute the 3.
	$0.5x - 6 = 1$	Combine like terms.
	$0.5x = 7$	Add 6 to both sides.
	$x = 14$	Divide both sides by 0.5, which is the same as multiplying by 2.
	$y = 0.1(14) - 2$	Substitute in $x = 14$ to the first equation.
	$y = 1.4 - 2 = -0.6$	The point of intersection is $(14, -0.6)$.

Explanations for Chapter 5 Practice Drill 6 - Number of Solutions

1.	Same slope, different y -intercepts	The two lines are already in $y = mx + b$ form, so we can see that they both have a slope of 2. They have different y -intercepts (-7 and 7), so they are two different, parallel lines. Parallel lines never cross, so this system has 0 solutions.
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$8y = -3x + 5$ $y = -\frac{3}{8}x + \frac{5}{8}$ $-8y = -3x + 5$ $y = \frac{3}{8}x - \frac{5}{8}$ $-\frac{3}{8}x + \frac{5}{8} = \frac{3}{8}x - \frac{5}{8}$	<p>Let's put each equation into $y = mx + b$ form to see the slope more easily. (If you're comfortable finding the slope from the standard form, you can do that as well.) Subtract $3x$ from both sides.</p> <p>Divide both sides by 8.</p> <p>Now do the same with the other equation. Subtract $3x$ from both sides.</p> <p>Divide both sides by -8. The lines have different slopes: the first line has a slope of $-\frac{3}{8}$, but the second line has a slope of $+\frac{3}{8}$. Therefore, the lines will cross at 1 point, and the system has exactly 1 solution.</p> <p>Find the point of intersection by setting the equations equal to each other.</p>
<p>2.</p> $\frac{5}{8} = \frac{6}{8}x - \frac{5}{8}$ $\frac{10}{8} = \frac{6}{8}x$ $\frac{5}{4} = \frac{3}{4}x$ $\frac{4}{3} \left(\frac{5}{4} \right) = x$ $\frac{5}{3} = x$ $3 \left(\frac{5}{3} \right) + 8y = 5$ $5 + 8y = 5$ $8y = 0$	<p>Add $\frac{3}{8}x$ to both sides.</p> <p>Add $\frac{5}{8}$ to both sides.</p> <p>Reduce the fractions.</p> <p>Multiply both sides by $\frac{4}{3}$ to isolate x.</p> <p>Substitute in $x = \frac{5}{3}$ to either of the original equations. I'm using the first one.</p> <p>Subtract 5 from both sides. $y = 0$ and the point of intersection is $\left(\frac{5}{3}, 0\right)$.</p>

$-y = -6x + 14$ $y = 6x - 14$ $-6y = -x + 14$ $y = \frac{1}{6}x - \frac{14}{6}$ $6x - 14 = \frac{1}{6}x - \frac{14}{6}$	<p>Let's put each equation into $y = mx + b$ form to see the slope more easily. (If you're comfortable finding the slope from the standard form, you can do that as well.) Subtract $6x$ from both sides.</p> <p>Multiply everything by -1. The slope is 6 and the y-intercept is -14.</p> <p>Now the other equation. Subtract x from both sides.</p> <p>Divide both sides by -6. The slope is -6. Since the lines have different slopes, they will intersect once, and have 1 solution.</p> <p>To find the point of intersection, set the two equations equal to each other.</p>
<p>3.</p> $36x - 84 = x - 14$ $35x - 84 = -14$ $35x = 70$ $x = 2$ $6(2) - y = 14$ $12 - y = 14$ $-y = 2$ $y = -2$	<p>Multiply both sides by 6 to get rid of the fractions.</p> <p>Subtract x from both sides.</p> <p>Add 84 to both sides.</p> <p>Divide both sides by 35.</p> <p>Substitute in $x = 2$ to either equation to find y. I'm using the first equation.</p> <p>Subtract 12 from both sides.</p> <p>Multiply both sides by -1. The point of intersection is $(2, -2)$.</p>

- $-3y = -2x + 1$
 $y = \frac{2}{3}x - \frac{1}{3}$
 $-12y = -8x + 4$
 $y = \frac{8}{12}x - \frac{4}{12}$
 $y = \frac{2}{3}x - \frac{1}{3}$
4. $4(2x - 3y = 1)$
 $8x - 12y = 4$
- Let's put each equation into $y = mx + b$ form to see the slope more easily. (If you're comfortable finding the slope from the standard form, you can do that as well.) Subtract $2x$ from both sides.
- Divide both sides by -3 . The slope is $\frac{2}{3}$ and the y -intercept is $-\frac{1}{3}$.
- Now the other equation. Subtract $8x$ from both sides.
- Divide both sides by -12 .
- Reduce each fraction. The slope is $\frac{2}{3}$ and the y -intercept is $-\frac{1}{3}$. Since the lines have both the same slope *and* the same y -intercept, both equations represent the same line, and they have infinite solutions.
- OR
- The second equation is just the first equation, times 4.
- Since both equations are the same, just one is times 4, the system has infinite solutions. The second equation doesn't give us any new information. It's the same as the first equation.

- $\frac{3}{5}y = -\frac{2}{3}x + \frac{1}{10}$
 $y = -\frac{10}{9}x + \frac{5}{30}$
5. $9y = -10x + 1.5$
 $y = -\frac{10}{9}x + \frac{1.5}{9}$
 $\frac{1.5}{9} = \frac{15}{90} = \frac{1}{6}$
- Let's put each equation into $y = mx + b$ form to see the slope more easily. (If you're comfortable finding the slope from the standard form, you can do that as well.) Subtract $\frac{2}{3}x$ from both sides.
- Multiply both sides by $\frac{5}{3}$ to isolate y . The slope is $-\frac{10}{9}$ and the y -intercept is $\frac{5}{30} = \frac{1}{6}$.
- Now the other equation. Subtract $10x$ from both sides to isolate y .
- Divide both sides by 9. The slope is $-\frac{10}{9}$, the same slope as the other equation, so the lines are either parallel or they're the same line. (Either no solutions or infinite solutions.)
- The y -intercept is $\frac{1}{6}$, the same as the y -intercept of the other equation. Both equations have the same slope and the same y -intercept, so they're the same line, and there are infinite solutions. (The second equation is just the first times 15.)

Explanations for Chapter 5 Practice Drill 7 - Quadratic Intersections

- $x^2 - 7 = 2x + 8$
 $x^2 - 2x - 7 = 8$
 $x^2 - 2x - 15 = 0$
 $(x - 5)(x + 3) = 0$
 $x = 5$ or $x = -3$
 $y = 2(5) + 8 = 18$
 $y = 2(-3) + 8 = -6 + 8 = 2$
- 1.
- To find the points of intersection, set the equations equal to each other.
- Since we have a quadratic, we want to set the equation equal to zero. Subtract $2x$ from both sides.
- Subtract 8 from both sides.
- Try to factor first. We need something that multiplies to -15 and adds to -2 , so the factors are -5 and 3 .
- Set each factor equal to zero to solve.
- Use either equation to find each y -coordinate by substituting in each x -coordinate we found. The linear equation is a bit easier, so I'm using that. One point of intersection is $(5, 18)$.
- The other point of intersection $(-3, 2)$.

- 2.
- | | |
|----------------------|--|
| $y = -12 + x$ | Isolate y in each equation first. For the linear equation, add x to both sides. |
| $y = 18 - x^2$ | In the quadratic equation, subtract x^2 from both sides. |
| $-12 + x = 18 - x^2$ | Now set the two equations equal to each other find the points of intersection. |
| $x^2 + x - 12 = 18$ | Add x^2 to both sides. |
| $x^2 + x - 30 = 0$ | Subtract 18 to both sides. |
| $(x + 6)(x - 5)$ | Try to factor the quadratic first. We need something that multiplies to -30 and adds to 1. So the factors are 6 and -5 . |
| $x = -6$ or $x = 5$ | Set each factor equal to zero and solve. |
| $y - (-6) = -12$ | Use either equation to find each y -coordinate by substituting in each x -coordinate we found. The linear equation is a bit easier, so I'm using that. |
| $y + 6 = -12$ | |
| $y = -18$ | Subtract 6 from both sides. One point of intersection is $(-6, -18)$. |
| $y - 5 = -12$ | Now use the other x -coordinate. |
| $y = -7$ | Add 5 to both sides. The other point of intersection is $(5, -7)$. |

- 3.
- | | |
|------------------------|---|
| $x^2 + 3x + 2 = x + 1$ | To find the points of intersection, set the two equations equal to each other. |
| $x^2 + 2x + 2 = 1$ | To solve the quadratic, we want to set it equal to zero. Subtract x from both sides. |
| $x^2 + 2x + 1 = 0$ | Subtract 1 from both sides. |
| $(x + 1)^2 = 0$ | Try to factor quadratic first. We need something that multiplies to 1 and adds to 2, which is 1 and 1. |
| $x = -1$ | Since there's only one squared factor, there's only one solution: $x = -1$. |
| $y = (-1) + 1 = 0$ | To find the y -coordinate, substitute in $x = -1$ to either equation. The linear one is a bit easier. The only point of intersection is $(-1, 0)$. |

- 4.
- | | |
|------------------------------|--|
| $y = -10x + 34$ | Isolate y in the linear equation but subtracting $10x$ from both sides. |
| $x^2 - 10x + 25 = -10x + 34$ | To find the points of intersection, set the two equations equal to each other. |
| $x^2 + 25 = 34$ | To solve the quadratic, we need to set the equation equal to zero. Add $10x$ from both sides. |
| $x^2 = 9$ | Actually, since the x terms canceled out, let's just subtract 25 from both sides. We could also still set it equal to zero, and we'll end up with the difference of squares: $x^2 - 9 = 0$, which factors to $(x - 3)(x + 3) = 0$. |
| $x = \pm 3$ | Since we have an x^2 , we have two possible solutions, the positive and the negative square root of 9. |
| $10(3) + y = 34$ | Substitute in each value for x to find the y values. Here is $x = 1$. |
| $y = 4$ | Subtract 30 from both sides. One solution is $(3, 4)$. |
| $10(-3) + y = 34$ | Substitute in $x = -1$ to find the other y -coordinate. |
| $-30 + y = 34$ | |
| $y = 64$ | Add 30 to both sides. The other point of intersection is $(-3, 64)$. |

- 5.
- $$x + 3 = x^2 + x - 4$$
- $$3 = x^2 - 4$$
- $$0 = x^2 - 7$$
- To find the points of intersection, set the two equations equal to each other.
- To solve the quadratic, we need to set the equation equal to zero. Subtract x from both sides.
- Subtract 3 from both sides. From here you have 3 options for solving: you can bring the 7 to other side and solve $x^2 = 7$, you can factor it as the difference of squares, $(x - \sqrt{7})(x + \sqrt{7})$, or you can use the quadratic formula, The quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, with $a = 1$, $b = 0$, and $c = 7$.
- $$x = \pm\sqrt{7}$$
- No matter how you solve it, you'll get the same 2 answers: positive and negative $\sqrt{7}$.
- $$y = \sqrt{7} + 3$$
- Now find the y -coordinates by substituting in each x -coordinate.. The linear equation is, as usual, easier to use. One point of intersection is $(\sqrt{7}, 3 + \sqrt{7})$.
- $$y = -\sqrt{7} + 3$$
- Now find other the y -coordinates by substituting in the other x -coordinate. The other point of intersection is $(-\sqrt{7}, 3 - \sqrt{7})$.

- 6.
- $$y = -3x + 4$$
- $$-3x + 4 = x^2 + 3x + 2$$
- $$4 = x^2 + 6x + 2$$
- $$0 = x^2 + 6x - 2$$
- Set the linear equation in terms of y by subtracting $3x$.
- Set the two equations equal to each other. To solve the quadratic, first we'll set it equal to zero.
- Add $3x$ to both sides.
- Subtract 4 from both sides. Ideally, we'd try to factor it now, but there's nothing that multiplies to -2 that also adds to 6. So, we'll have to use the quadratic formula, with $a = 1$, $b = 6$, and $c = -2$.
- $$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-2)}}{2(1)}$$
- The quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- $$x = \frac{-6 \pm \sqrt{36 + 8}}{2}$$
- $$x = \frac{-6 \pm \sqrt{44}}{2}$$
- $$x = \frac{-6 \pm \sqrt{4}\sqrt{11}}{2}$$
- Break apart $\sqrt{44}$ using $44 = 4 \times 11$.
- $$x = \frac{-6 \pm 2\sqrt{11}}{2}$$
- $$x = -3 + \sqrt{11} \text{ or } x = -3 - \sqrt{11}$$
- Break apart the \pm into two separate answers. This question only wants the values for x , so we don't have to plug those back in to find y .

1.5 Chapter 6 Ratios and Rates Answers

Explanations for Chapter 6 Drill 1 - Ratios

-
1. $\frac{a}{b} = \frac{2}{9}$ Set up the ratio.
- $\frac{18}{b} = \frac{2}{9}$ Insert $a = 18$.
- $2b = 9(18)$ Cross multiply.
- $2b = 162$
- $b = 81$ Divide both sides by 2.
- OR
- $\frac{18}{b} = \frac{2}{9}$ Find the multiplier: $2 \times \underline{9} = 18$, so $b = 9 \times \underline{9} = 81$
-
2. $\frac{4 \text{ lions}}{3 \text{ tigers}} = \frac{x \text{ lions}}{12 \text{ tigers}}$ Set up the ratio.
- $\frac{4}{3} = \frac{x}{12}$
- $4(12) = 3x$ Cross multiply.
- $48 = 3x$ Simplify.
- $16 = x$ Divide both sides by 3.
- OR
- $\frac{4}{3} = \frac{x}{12}$ Find the multiplier. $3 \times \underline{4} = 12$. So the top and bottom are multiplied by 4 to make the fraction on the right, and $4 \times \underline{4} = 16$
-
3. $\frac{4 \text{ lions}}{3 \text{ tigers}} = \frac{12 \text{ lions}}{x \text{ tigers}}$ Set up the ratio.
- $\frac{4}{3} = \frac{12}{x}$
- $4x = 3(12)$ Cross multiply.
- $4x = 36$ Simplify.
- $x = 9$ Divide both sides by 4.
- OR
- $\frac{4}{3} = \frac{12}{x}$ Find the multiplier. $4 \times \underline{3} = 12$. So the top and bottom are multiplied by 3 to make the fraction on the right, and $3 \times \underline{3} = 9$.
-
4. $\frac{1 \text{ green}}{5 \text{ red}} = \frac{10 \text{ green}}{x \text{ red}}$ Set up the ratio.
- $\frac{1}{5} = \frac{10}{x}$
- $x = 5(10)$ Cross multiply.
- $x = 50$ Simplify.
- OR
- $\frac{1}{5} = \frac{10}{x}$ Find the multiplier. $1 \times \underline{10} = 10$. So the top and bottom are multiplied by 10 to make the fraction on the right, and $5 \times \underline{10} = 50$.
-

-
5. $\frac{x}{y} = \frac{2}{7}$ Set up the ratio.
- $\frac{140}{y} = \frac{2}{7}$ Insert $x = 140$
- $7(140) = 2y$ Cross multiply.
- $980 = 2y$ Simplify.
- $490 = y$ Divide both sides by 2.
- OR
- $\frac{140}{y} = \frac{2}{7}$ Find the multiplier. $2 \times \underline{70} = 140$. So the top and bottom are multiplied by 70 to make the fraction on the left, and $7 \times \underline{70} = 490$.
-

6. $\frac{2 \text{ hot dogs}}{1 \text{ hamburgers}} = \frac{60 \text{ hot dogs}}{x \text{ hamburgers}}$ Set up the ratio.
- $\frac{2}{1} = \frac{60}{x}$
- $2x = 1(60)$ Cross multiply.
- $2x = 60$ Simplify.
- $x = 30$ Divide both sides by 2.
- OR
- $\frac{2}{1} = \frac{60}{x}$ Find the multiplier. $2 \times \underline{30} = 60$. So the top and bottom are multiplied by 30 to make the fraction on the right, and $1 \times \underline{30} = 30$.
-

7. $\frac{20 \text{ kathas}}{1 \text{ bigha}} = \frac{x \text{ kathas}}{20 \text{ bigha}}$ Set up the ratio.
- $\frac{20}{1} = \frac{x}{20}$
- $x = 20(20)$ Cross multiply.
- $x = 400$ Simplify.
- OR
- $\frac{20}{1} = \frac{x}{20}$ Find the multiplier. $1 \times \underline{20} = 20$. So the top and bottom are multiplied by 20 to make the fraction on the right, and $20 \times \underline{20} = 400$.
-

8. $\frac{1 \text{ inch}}{3 \text{ miles}} = \frac{4 \text{ inches}}{x \text{ miles}}$ Set up the ratio.
- $\frac{1}{3} = \frac{4}{x}$
- $x = 12$ Cross multiply.
- OR
- $\frac{1}{3} = \frac{4}{x}$ Find the multiplier. $1 \times \underline{4} = 4$. So the top and bottom are multiplied by 4 to make the fraction on the right, and $3 \times \underline{4} = 12$.
-

9. $\frac{3 \text{ cm}}{1 \text{ foot}} = \frac{27 \text{ cm}}{x \text{ feet}}$ Set up the ratio.
 $\frac{3}{1} = \frac{27}{x}$
 $3x = 1(27)$ Cross multiply.
 $x = 9$ Divide both sides by 3
 OR
 $\frac{3}{1} = \frac{27}{x}$ Find the multiplier. $3 \times \underline{9} = 27$. So the top and bottom are multiplied by 30 to make the fraction on the right, and $1 \times \underline{9} = 9$.
-

10. $\frac{3 \text{ defective}}{400 \text{ chips}} = \frac{x \text{ defective}}{8,000 \text{ chips}}$ Set up the ratio.
 $\frac{3}{400} = \frac{x}{8,000}$
 $400x = 3(8,000)$ Cross multiply.
 $400x = 24,000$ Simplify.
 $x = 60$ Divide both sides by 400.
 OR
 $\frac{3}{400} = \frac{x}{8,000}$ Find the multiplier. $400 \times \underline{20} = 8,000$. So the top and bottom are multiplied by 20 to make the fraction on the right, and $3 \times \underline{20} = 60$.

Explanations for Chapter 6 Practice Drill 2 - Rates

1. $15.75 = 0.35 \times \text{weight}$ The rate here is \$0.35 per kg (dollars per kilogram).
 $45 \text{ kg} = \text{weight}$ Divide both sides by 0.35. Notice that the dollars on the left canceled out, leaving just kilograms: the unit we want for weight.
-

2. $9 \text{ hours} - 1 \text{ hour break} = 8 \text{ hours driving}$ Start by finding the total time the family spent driving.
 $432 = R(8)$ $D = R \times T$
 $54 \text{ mph} = R$ Divide both sides by 8 hours.
-

3. $20,250 - 4,130 = 16,120$ Start by finding out how much more water we need to put in the pool by taking the total gallons and subtracting the amount already in the pool.
 $16,120 = 20T$ $D = R \times T$
 $806 \text{ minutes} = T$ Divide both sides by 20.
-

4. $d = 55t$ The question gives us a function to find the distance, in terms of time.
 $d = 55(3)$ Since we are told she has driven for 3 hours, $t = 3$.
 $d = 165$ Done.
-

-
5. $\frac{1}{3} = R(2)$
 $\frac{1}{6} = R$
 $1 = \frac{1}{6}(T)$
 $6 = T$
- $W = R \times T$. He's painted $\frac{1}{3}$ of the wall, so $W = \frac{1}{3}$, and $T = 2$
- Divide both sides by 2. Now we know the rate per hour, let's figure out how long it takes to paint one entire wall.
- Plugging in $W = 1$ and $R = \frac{1}{6}$.
- Divide both sides by $\frac{1}{6}$.
- OR
- If he paints $\frac{1}{3}$ of the wall in 2 hours, it's going to take him 3 times longer, $\frac{1}{3} \times 3 = 1$, to paint the entire thing.
-
6. 50 miles per hour for x hours
 $D = 50(1)$
 $290 - 50 = 240$ miles
 $240 = 40(T)$
 $6 = T$
- Start with the first part of Lily's journey. Since $x = 1$, we can find out how many total miles she drove at first.
- $D = R \times T$, so the distance is 50 miles. Well, that makes sense. 50 miles per hour, she drove 1 hour, so that's 50 miles. Checks out.
- Since she drove for 50 miles at first, she drove the remaining 240 miles at 40 mph
- $D = R \times T$
- Divide both sides by 40.
-
7. $120 = R(3)$
 $40 = R$
 $W = 40(10)$
 $W = 400$
- $W = R \times T$
- Divide both sides by 3
- Use that rate in $W = R \times T$, with $T = 10$
-
8. $D = 2(60) = 120$ miles
 $D = 3(50) = 150$ miles
 $120 \text{ miles} + 150 \text{ miles} = 270 \text{ miles}$
 $2 \text{ hours} + 3 \text{ hours} = 5 \text{ hours}$
 $270 = R(5)$
 $54 = R$
- To find the average speed, we need to find the total distance and the total time. Use $D = R \times T$ with each portion of the journey.
- Find the total distance and the total time.
- $D = R \times T$
- Divide both sides by 5
-
9. $60\% - 50\% = 10\%$
 $10\% = R(20)$
 $0.5\% = R$
 $100\% - 60\% = 40\%$
 $40\% = 0.5\%(T)$
 $80 = T$
 $20(4) = 80$
- Since the painter had already painted half the fence (50%), by the time they've painted 60% of the fence they've painted an additional 10% of the fence.
- $W = R \times T$. $W = 10\%$ of the fence, which it took $T = 20$ minutes to paint.
- So the painter is painting 0.5% of the fence per minute.
- He still has 40% of the fence left to paint
- $W = R \times T$. $W = 40\%$ of the fence, $R = 0.5\%$ of the fence per minute.
- Divide both sides by 0.5%
- OR
- Since it took 20 minutes to paint 10% of the fence, and he still has to paint 40% of the fence, it's going to take 4 times longer to finish. Each 10% is 20 minutes, so it'll take 20(4) minutes to finish painting.

	$H + L = 330$	This, which combines rates with systems of equations, is one of the hardest rate questions you'll see on the SAT, and it's pretty rare. We know that Artoun drove a total of 330 miles on the highway and on local roads. For now, let's make up some variables so we can write an equation in terms of distance. Remember, if there's ever something you don't know in a word problem, assign it a variable, so you can solve for it later. We'll call the total distance he drove on highways H and the total distance he drove on local roads L .
	$H = 45(T_H)$	$D = R \times T$. We don't know the time he spend on the highway, so we'll call it T_H
	$L = 30(T_L)$	$D = R \times T$. The same equation as above, but now for local roads.
	$T_H + T_L = 9$	He drove for a total of 9 hours.
		So, we have 4 equations, and 4 variables. (See Chapter 5: Systems of Equations on page 1, if the following steps don't make sense.)
10.	$T_H = \frac{H}{45}$	Isolate T_H .
	$T_L = \frac{L}{30}$	Isolate T_L .
	$\frac{H}{45} + \frac{L}{20} = 9$	Insert our equations for T_H and T_L into the $T_H + T_L = 9$ equation, for total time.
	$2H + 3L = 810$	Multiply both sides by 90 to get rid of the fractions. (For more information, see the Fractions section on page ?? of Chapter 2: Solving Equations)
	$L = 330 - H$	Isolate L in the first equation, so that we'll still have H , the total distance driven on highways, in the final equation.
	$2H + 3(330 - H) = 810$	Insert $L = 330 - H$ into our equation.
	$2H + 990 - 3H = 810$	Distribute.
	$-H = -180$	Combine like terms and subtract 990 from both sides.
	$H = 180$	180 total miles.

Explanations for Chapter 6 Practice Drill 3 - Dimensional Analysis

	$\frac{6 \text{ miles}}{\text{gallon}}$	Set up the ratio.
	$\frac{6 \text{ miles}}{\text{gallon}} = \frac{x \text{ miles}}{240 \text{ gallon}}$	Add the information given.
1.	$x = 6(240) = 1440$	Cross multiply.
		OR
	$\frac{6 \text{ miles}}{\text{gallon}}$	To cancel out the gallons, just leaving miles, we can multiply by gallons.
	$\frac{6 \text{ miles}}{\text{gallon}} \times \frac{240 \text{ gallons}}{1} = 1440 \text{ miles}$	

-
2. $\frac{6 \text{ miles}}{\text{gallon}}$ Set up the ratio.
- $\frac{6 \text{ miles}}{\text{gallon}} = \frac{240 \text{ miles}}{x \text{ gallons}}$ Add the information given.
- $6x = 240$ Cross multiply.
- $x = 40$ Divide both sides by 6.
- OR
- $\frac{6 \text{ miles}}{\text{gallon}}$ Since want to end up with gallons, we'll take the miles and divide by the miles per gallon. Since division by a fraction is the same as multiplying by the reciprocal of the fraction, we'll multiply by gallons per mile.
- $\frac{240 \text{ miles}}{1} \times \frac{1 \text{ gallon}}{6 \text{ miles}} = 40 \text{ gallons}$
-
3. $\frac{\text{wolves}}{\text{square mile}}$ We need density, in wolves per square mile.
- $\frac{10,000 \text{ wolves}}{660,000 \text{ square miles}}$ Set up the ratio.
- $\frac{1 \text{ wolf}}{66 \text{ square miles}} \approx 0.015 \frac{\text{wolves}}{\text{square mile}}$ Simplify.

Explanations for Chapter 6 Practice Drill 4 - Unit Conversions

-
1. 60 miles per hour = 60 miles in 1 hour Break up the rate. We need miles per minute.
- 1 hour = 60 minutes Convert hours into minutes.
- $60 = R(60)$ $D = R \times T$
- $R = 1 \text{ mile per minute}$
-
2. $\frac{1013 \text{ mb}}{760 \text{ mm Hg}} = \frac{840 \text{ mb}}{x \text{ mm Hg}}$ Set up the ratio
- $1013x = 760(840)$ Cross multiply.
- $1,013x = 638,400$
- $x = 630.207... = 630$ Divide both sides by 1013 and round to the nearest whole number.
-
3. 150 meters in 12 seconds First we can find his rate with $D = R \times T$
- $15 = R(12)$
- $R = \frac{1.25 \text{ meters}}{\text{second}}$ Divide both sides by 12
- $6(60) = 360 \text{ seconds}$ Convert 6 minutes into seconds.
- $D = 1.25(360)$ Use the rate, 1.25 m/s, that we found earlier.
- $D = 450$
-

1.5 Chapter 6 Ratios and Rates Answers and Explanations

4. $0.12(400) = 48$ ounces Since the rate is in terms of ounces, figure out how many total ounces are required for 400 cupcakes.
- $\frac{1 \text{ pounds}}{16 \text{ ounces}} = \frac{x \text{ pounds}}{48 \text{ ounces}}$ Convert ounces to pounds.
- $16x = 48$ Cross multiply.
- $x = 3$ Divide both sides by 16.
-

5. $\frac{1 \text{ kilogram}}{2.205 \text{ pounds}} = \frac{x \text{ kilograms}}{72 \text{ pounds}}$ Set up the ratio.
- $2.205x = 72$ Cross multiply.
- $x = 32.6530\dots = 32.65$ Divide both sides by 2.205 and round to the nearest hundredth.
-

6. 343 meters per 1 second Break up the rate. We need to convert the seconds into minutes and the meters into feet.
- $\frac{60 \text{ seconds}}{1 \text{ minute}} = \frac{1 \text{ second}}{x \text{ minutes}}$ Convert the seconds to minutes.
- $60x = 1$
- $x = \frac{1}{60}$ minute
- $\frac{1 \text{ meter}}{3.28 \text{ feet}} = \frac{343 \text{ meters}}{x \text{ feet}}$ Set up the ratio to convert meters to feet.
- $3.28(343) = x$ Cross multiply.
- $x = 1,125.04$ feet
- $1,125.04 = R \frac{1}{60}$ $D = R \times T$, using meters and minutes.
- 67,502.4 Multiply both sides by 60.
-

7. 45 miles in 1 hour Break up the rate. We need feet and minutes.
- $\frac{1 \text{ mile}}{5,280 \text{ feet}} = \frac{45 \text{ miles}}{x \text{ feet}}$ Convert miles to feet.
- $x = 5280(45) = 237,600$ feet
- 1 hour = 60 minutes Convert hours to minutes.
- $237,600 = R(60)$ Use $D = R \times T$ to find the rate in feet per minute.
- $3,960 = R$
- $D = 3,960(15)$ Use $D = R \times T$ again, but now use the rate in terms of feet and minutes.
- $D = 59,400$ feet
-

8. $\frac{1 \text{ ton}}{2,000 \text{ pounds}} = \frac{5.3 \text{ tons}}{x \text{ pounds}}$ Set up the ratio.
- $x = 10,600$ Cross multiply.
-

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9. 4 yards = 12 feet Since the question asks for the volume in cubic feet, convert everything to feet before calculating the volume.
3 yards = 9 feet Use 1 yard = 3 feet.
8 inches = $\frac{2}{3}$ foot Use 12 inches = 1 foot.
 $V = lwh = 12(9)\left(\frac{2}{3}\right)$ Use the volume formula.
 $V = 72$ cubic feet Simplify.
-
10. 0.5 miles = 80 mph $\times T$ First find how long, in hours, it takes for a bald eagle to travel half a mile by using $D = R \times T$
 $T = 0.00625$ hours Now that we have the answer in hours, convert to seconds.
 $0.00625(60) = 0.375$ minutes Use 1 hour = 60 minutes.
 $0.375(60) = 22.5$ seconds Use 1 minute = 60 seconds.

1.6 Chapter 7 Percents Answers

Explanations for Chapter 7 Practice Drill 1 - Percent Fundamentals

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1. $0.60(350) = 210$ The whole number is 350, and we want a part of that: 60% of that, to be precise. $60\% = \frac{60}{100}$ or 0.60, so 60% of 350 is just 60% times 350.
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2. $0.35(260) = 91$ 35% of the whole number 260, is 0.35 times 260.
-
3. $0.85(40) = 34$ 85% of the total number of fish are koi fish, so the koi represent 85% of the 40 fish, or 0.85 times 40.
-
4. $1.40(60) = 84$ 140% of a number is just the number times 1.4. (100% is the number times 1, or the same number, and 40% of a number is just the number plus another 40% of the number.)
-
5. $\frac{90}{240} = \frac{x}{100}$ Put what we know in the equation $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$. The part is 90 and the whole is 240.
 $0.375 = \frac{x}{100}$ Either divide on the left or cross multiply. Here, I'm dividing with my calculator.
 $37.5 = x$ Multiply both sides by 100. 37.5% of 240 is 90.
-
6. $70 + 55 = 125$ First, let's find out how many people were asked by adding up the number of people who preferred Picture A plus the number of people who preferred Picture B.
 $\frac{70}{125} = \frac{x}{100}$ Now we know the part, 70, and the whole, 125, so put them in the equation $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$.
 $0.56 = \frac{x}{100}$ Either cross multiply or divide on the left. I'm using my calculator to divide on the left, because I'm being a bit lazy.
 $56 = x$ Multiply both sides by 100. 56% of the people preferred Picture A.
-
7. $17 + 4 = 21$ Let's find the number of basketballs first. There are 31 basketballs, which is just a part of the total number of basketballs.
 $17 + 14 + 4 = 35$ The total number of balls includes the 14 dodgeballs. Now we have the part and the total, so we can find the percent.
 $\frac{21}{35} = \frac{x}{100}$ Put those numbers in the equation $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$.
 $0.6 = \frac{x}{100}$ Divide on the left or cross multiply. Me? I'm dividing on the left.
 $60 = x$ Multiply both sides by 100. 60% of the balls are basketballs.
-
8. $\frac{40}{x} = \frac{32}{100}$ 32% of some larger number is 40. So 40 is the *part*, we don't know the *whole*, x , and the percent is 32. Put all of that in the equation $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$.
 $4000 = 32x$ Cross multiply.
 $125 = x$ Divide both sides by 32.

-
9. $\frac{7}{x} = \frac{17.5}{100}$
 $700 = 17.5x$
 $40 = x$
- Those 7 two-bedroom apartments are only *part* of the total number of apartments. 17.5% of the apartments, in fact. Put all of that in the equation $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$.
- Cross multiply.
- Divide both sides by 17.5. Those 7 two-bedroom apartments are 17.5% of the 40 apartments in the building.
-
10. $\frac{24.6}{x} = \frac{30}{100}$
 $2460 = 30x$
 $82 = x$
- The 24.6 mL of stuff is only *part* of the total stuff, it's only 30% of the stuff, but we need to know the whole amount of stuff. So that's what we put into the equation $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$.
- Cross multiply.
- Divide both sides by 30. The 24.6 mL is 30% of the total 80 mL of stuff.

Explanations for Chapter 7 Practice Drill 2 - Percent Change

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1. $11.5 - 9.2 = 2.3$
 $\frac{2.3}{9.2} = \frac{x}{100}$
 $0.25 = \frac{x}{100}$
 $25 = x$
- First, find the difference. Moujan walked 2.3 hours more this month than last month.
- We want to know the percent increase compared to last month, so that's 9.2 is the original number, underneath the fraction.
- Divide on the left or cross multiply.
- Moujan increased her number of hours walked by 25%.
-
2. $246.50 - 212.50 = 34.00$
 $\frac{34}{212.50} = \frac{x}{100}$
 $0.16 = \frac{x}{100}$
 $16 = x$
- Find the difference between the two bills. In June, Cameron paid \$34.00 more than he did in May.
- We want to know the percent increase compared to last May, so that's the original number, underneath the fraction.
- Either cross multiply or divide. I am dividing.
- Multiply both sides by 100. Cameron's bill increased by 16%.
-
3. $5,150 - 4,300 = 850$
 $\frac{850}{4300} = \frac{x}{100}$
 $0.1976 \approx \frac{x}{100}$
 $19.8 = x$
- Find the difference in the number of subscribers from 2020 to 2021.
- We want to know the percent increase compared to 2020, so that's the original number, underneath the fraction.
- Divide.
- Multiply both sides by 100 and round to the nearest tenth.
-
4. $20,995 - 16,585 = 4,410$
 $\frac{4,410}{20,995} = \frac{x}{100}$
 $0.21005 = \frac{x}{100}$
 $21 = x$
- Find the difference in price between the year it was released and a year later.
- We want to know the percent decrease compared to the price when the car was released, so that's the original number, underneath the fraction.
- Divide.
- Multiply by 100 and round to the nearest tenth.

- $8(10) = 80$ The current area is $length \times width$, which is 80 inches².
- $10(12) = 120$ The new length and width (or height and width, whatever) are each 2 inches longer. So, instead of 8 inches wide, it's now 10 inches wide, and instead of 10 inches long, it's now 12 inches long. The new area is 120 inches².
5. $120 - 80 = 40$ The difference in areas is 40 inches².
- $\frac{40}{80} = \frac{x}{100}$ We want to know the percent increase compared to the original area, so that's the number underneath the fraction.
- $0.5 = \frac{x}{100}$ Divide.
- $50 = x$ Multiply both sides by 100. The area of the rectangle increased by 50%.

Explanations for Chapter 7 Practice Drill 3 - After a Percent Change

1. $100\% - 20\% = 80\%$ The price of the book has been discounted by 20%, so the new price is only 80% of the original price.
- $0.8(24) = 19.2$ Find 80% of \$24. The new price is \$19.20.
-
2. $100\% + 15\% = 115\%$ We want a number 15% greater than 120. So, the new number is 115% of 120.
- $1.15(120) = 138$ Since $115\% = 1.15$, multiply 120 by 1.15. 138 is 15% greater than 120.
-
3. $100\% + 50\% = 150\% = 1.5$ The new edition is 150% the length of the previous edition, or 1.5 times the previous edition.
- $1.5(380) = 570$ Multiply. The new edition is 50% greater, and has 570 pages.
-
4. $100\% + 18\% = 118\% = 1.18$ An increase of 18% means the new weight of the puppy is 1.18 times the old weight.
- $1.18x$ Since the old weight is x , the new weight, 18% greater, is $1.18x$.
-
5. $220 - 160 = 60$ Find the percent increase from 2018 to 2019 first. The gardening company had 60 more clients in 2019 than in 2018.
- $\frac{60}{160} = \frac{x}{100}$ Find the percent increase by putting the change, 60 clients, over the original number, the 160 clients in 2018.
- $0.375 = \frac{x}{100}$ Divide.
- $37.5 = x$ The number of clients increased by 37.5% in 2019.
- $2(37.5) = 75$ In 2020, the number of clients increased by double the percent increased from 2018 to 2019: so, it increased by double 37.5%, or 75%.
- $100\% + 75\% = 175\% = 1.75$ The number of clients in 2020 is the number of clients in 2019 times 1.75.
- $1.75(220) = 385$ In 2019, there were 220 clients, so let's find 175% of 220 to find the number of clients in 2020. Multiply it out, and there were 385 clients in 2020.

Explanations for Chapter 7 Practice Drill 4 - Before a Percent Change

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1. $100\% - 20\% = 80\%$
 $(0.80)x = 520$
 $x = 650$
- The new price of the bike is 80%, or 0.80 times, the old price of the bike.
80% of the old price is the new price.
Divide both sides by 0.8. The original price of the bike, before the discount, was \$650.
-
2. $100\% + 15\% = 115\% = 1.15$
 $(1.15)x = 89.70$
 $x = 78$
- The price after the surcharge is 1.15 times the price before the increase.
115% of the price before the surcharge is the final price.
Divide both sides by 1.15. The price before the surcharge was \$78.
-
3. $100\% + 25\% = 125\% = 1.25$
 $(1.25)x = 35$
 $x = 28$
- The number of people in the morning class was 1.25 times the number in the afternoon class.
The number of people in the afternoon class, x , times 1.25 is the number of people in the morning class.
Divide both sides by 1.25. There were 28 people in the afternoon class.
-
4. $100\% + 17.5\% = 117.5\% = 1.175$
 $(1.175)x = 531,100$
 $x = 452,000$
- An increase of 17.5% means that the current price times 1.175 is the new price. (Remember that we convert from a percent to decimal by moving the decimal two digits to the left, so 117.5% becomes 1.175.)
The current price, x , times 1.175 gives us next year's price of \$531,100.
Divide both sides by 1.175. The median price this year is \$452,000.
-
5. $100\% - 30\% = 70\% = 0.70$
 $(0.70)y = x$
 $y = \frac{x}{0.7}$
- The discounted price is 70% of the original price.
The discounted price, x , is the original price, y , times 0.7.
Solve for x . (This is the same as $\frac{10x}{7}$, so if you put that, that's fine as well.)

1.7 Chapter 8 Statistics Answers

Explanations for Chapter 8 Practice Drill 1 - Median and Range

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| 1. | <p>8, 10, 15, 100, 172</p> <p>Range = $172 - 8 = 164$</p> <p>Median = 15</p> | <p>Start by putting the list in order.</p> <p>The range is the largest number minus the smallest.</p> <p>The median is the middle number. There are 5 numbers, so the median is the 3rd number, 15.</p> |
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| 2. | <p>Range = $80 - 10 = 70$</p> <p>Median = 60</p> | <p>The range is the largest number (the whisker on the right) minus the smallest number (the whisker on the left).</p> <p>The median in a box plot is the middle line of the box. 50% of the numbers are above that line, and 50% are below.</p> |
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| 3. | <p>Range = $5 - 1 = 4$</p> <p>$5 + 2 + 5 + 4 + 3 = 19$</p> <p>$19 \div 2 = 9.5$</p> | <p>The range is the largest value minus the smallest value. Notice that we don't care about the frequency of the numbers, just what they are.</p> <p>We can find the median by first finding how many total values there are. Add up all the frequencies to find that there are 19 total numbers.</p> <p>Divide that by 2 and round up. The 10th number is our median. The first $5 + 2 = 7$ numbers are going to be 1s and 2s. The next 5 numbers (the 8th, 9th, 10th, 11th, and 12th numbers in our list) are all 3s, so the median is 3.</p> <p>OR</p> <p>Write out the list. Yes, it's annoying, but it's doable. It took me less than 30 seconds to type out that list, so you can definitely write it out on the test. There are 19 total numbers, so the median is the 10th number on the list, which will have 9 numbers to the left and 9 numbers to the right. The 9th number is 3.</p> |
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| 4. | <p>Range = $264 - 163 = 101$</p> <p>163, 169, 172, 180, 207, 224, 235, 251, 264</p> | <p>Find the largest and smallest values and subtract them to find the range.</p> <p>To find the median, put the numbers in order. Since there are 9 numbers, the median is the 5th number, 207. (You can also find it directly from the graph, by finding which point has 4 points lower than it and 4 points higher than it: 207.)</p> |
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| 5. | <p>Range = $6 - 0 = 6$</p> <p>Median = 4</p> | <p>To find the range of the number of fire trucks, subtract the smallest number of fire trucks (0) from the largest number of fire trucks (6).</p> <p>To find the median, find the middle number. Since there are 13 towns, the 7th number will be the median, with 6 numbers to the left and 6 to the right. Since the number of towns with 0, 1, 2, or 3 fire trucks adds up to $3 + 1 + 2 + 0 = 6$, the median is the town with 4 fire trucks. (Notice that there's also 6 numbers after it: 5 towns with 5 trucks and 1 town with 6 trucks.)</p> <p>OR</p> <p>Write out the number of fire trucks as a list. There were 3 towns with 0 trucks, 1 town with 1 truck, and so on. The median number is the 7th number in line, which is 4.</p> |
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Explanations for Chapter 8 Practice Drill 2 - Mean

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1. $5 + 10 + 12 + 13 + 20 = 60$ Since $Average = \frac{Total}{Number}$, to find the average, start by finding the total.
 $60 \div 5 = 12$ Divide the total by the number of numbers, 5, to get the average.
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2. $6(12) = 72$ Start by finding the total. $Total = Average \times Number$ We know the average is 12, and we have 6 numbers.
 $x + 7 + 11 + 14 + 15 + 21 = 72$ Now we know that all 6 numbers added together equals 72.
 $x + 68 = 72$
 $x = 4$ Subtract 68 from both sides.
-
3. $4(82) = 328$ Start by finding Kendahl's total score on all 4 tests. $Total = Average \times Number$
 $5(84) = 420$ To get an 84 on 5 tests, Kendahl needs to have a total of 420 points.
 $420 - 328 = 92$ The total from 4 tests to 5 tests changed by 92 points, so Kendall needs to get at least 92 on the 5th test to get an average of 84.
-
4. $15 + 18 + x$ Since $Average = \frac{Total}{Number}$, start by finding the total.
 $33 + x$
 $\frac{33 + x}{3}$ Divide the total by the number of weeks, 3, to find the average miles per week.
-
5. $a = 2$ Let's try out a number. As (almost) always, we can try $a = 2$. In that case, our consecutive integers are 2, 3, 4, 5, and 6.
 $2 + 3 + 4 + 5 + 6 = 20$ Find the total.
 $\frac{20}{5} = 4$ Find the average of the 5 numbers. Since $a = 2$, 4 is $a + 2$.
 For consecutive numbers, mean = median By the way, for any set of consecutive numbers, the mean will always equal the median. So you can find the median and just use that. (This is actually true for any arithmetic sequence, where the numbers always increase or decrease by the same amount.)
 OR
 $a, a + 1, a + 2, a + 3, a + 4$ If our first integer is a , the integer after is $a + 1$, and so on.
 $a + a + 1 + a + 2 + a + 3 + a + 4$ Find the total.
 $5a + 10$
 $\frac{5a + 10}{5}$ Divide by the number of integers, 5.
 $a + 2$ Simplify.

1.8 Chapter 9 Angles, Triangles, and Trigonometry Answers

Explanations for Chapter 9 Practice Drill 1 - Isosceles Triangles

1.	$a = c$ $c = 40$	Angles opposite equal sides are equal.
2.	$b = c$ $180 = a + b + c$ $180 = 40 + c + c$ $140 = 2c$ $70 = c$	Angles opposite equal sides are equal. 180 degrees in a triangle. Use $b = c$ and $a = 40$. Combine like terms, subtract 40 from both sides. Divide both sides by 2.
3.	$AB = AC$ $AC = 8$	Sides opposite equal angles are equal. Use $AB = 8$.
4.	$AC = BC$ $AC = 11$	Sides opposite equal angles are equal. Use $BC = 11$.
5.	$c = b$ $180 = a + b + c$ $180 = 50 + b + b$ $130 = 2b$ $65 = b$	Angles opposite equal sides are equal. 180 degrees in a triangle. Use $c = b$ and $a = 50$. Combine like terms, subtract 50 from both sides. Divide both sides by 2.
6.	$a = c$ $180 = a + b + c$ $180 = a + 90 + a$ $90 = 2a$ $45 = a$	Angles opposite equal sides are equal. 180 degrees in a triangle. Use $c = a$ and $b = 90$ Combine like terms, subtract 90 from both sides. Divide both sides by 2.
7.	$b = a$ $180 = a + b + c$ $180 = 2c + 2c + c$ $180 = 5c$ $36 = c$ $a = 2(36) = 72$	Angles opposite equal sides are equal. 180 degrees in a triangle. Use $a = b = 2c$ Combine like terms. Divide both sides by 5. Use $a = b = 2c$.

	$BC = AC$	Sides opposite equal angles are equal.
8.	$4x - 12 = 3x - 8$	Use $BC = 4x - 12$, and $AC = 3x - 8$
	$x - 12 = -8$	Subtract $3x$ from both sides.
	$x = 4$	Add 12 to both sides.

	$180 = a + b + c$	180 degrees in a triangle, always a good place to start if you're stuck on a tough triangle question.
	$180 = (9x - 2) + (7x - 1) + (5x + 15)$	Use $a = 9x - 2$, $b = 7x - 1$, and $c = 5x + 15$.
	$180 = 21x + 12$	Combine like terms.
	$168 = 21x$	Subtract 12 from both sides.
	$8 = x$	Divide both sides by 21.
9.	$a = 9(8) - 2$	Find a using $a = 9x - 2$.
	$a = 70$	
	$b = 7(8) - 1$	Find b using $b = 7x - 1$.
	$b = 55$	
	$c = 5(8) + 15$	Find c using $c = 5x + 15$.
	$c = 55$	So $b = c$, which means that $AB = AC$, since sides opposite equal angles are equal.
	$AC = AB = 5$	

	$AB = BC$	Sides opposite equal angles are equal.
	$x^2 = 3x - 10$	Use $AB = x^2 - 10$, and $BC = 3x$.
	$x^2 - 3x - 10 = 0$	Solve the quadratic. For more information on solving quadratics, check out Chapter 10: Quadratics, on page 1.
10.	$(x - 5)(x + 2) = 0$	Find something that adds to -3 and multiplies to -10 : -5 and 2 .
	$x = 5$ or -2	Either $(x - 5) = 0$ or $(x + 2) = 0$. Since -2 gives a negative side length for BC , we'll just use $x = 5$
	$BC = 3(5) = 15$	Use $BC = 3x$.

Explanations for Chapter 9 Practice Drill 2 - Parallel Lines

1.	$a = 45$	The vertical angle for the 45° is an alternate interior angle to a .
2.	$180 = b + 45$	180° in a line.
	$135 = b$	Subtract 45 from both sides.
3.	$c = 55$	c is an alternate interior angle to 55° .

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| 4. | $180 = c + d$
$180 = 55 + d$
$125 = d$ | 180° in a line.
Use $c = 55$ from the previous problem.
Subtract 55 from both sides. |
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| 5. | $180 = a + e + 55^\circ$
$180 = 45 + e + 55$
$180 = 100 + e$
$80 = e$ | The vertical angle to e is in a line with a and 55 . 180° in a line.
Use $a = 45$ from the first problem.
Combine like terms.
Subtract 100 from both sides |
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| 6. | $v = 35$ | The vertical angle for the 35° is an alternate interior angle to v . |
| <hr/> | | |
| 7. | $180 = 35 + 80 + w$
$180 = 115 + w$
$65 = w$ | The vertical angles for the 35° and 80° are part of the same triangle as w .
Combine like terms.
Subtract 115 from both sides. |
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| 8. | $180 = w + x$
$180 = 65 + x$
$115 = x$ | The alternate interior angle to w is supplemental to x .
Use $w = 65$ from the previous problem.
Subtract 65 from both sides. |
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| 9. | $180 = y + w + 40$
$180 = y + 65 + 40$
$180 = y + 105$
$75 = y$ | The corresponding angle to y (the vertical angle to the alternate interior angle) is part of the triangle on the top right.
Use $w = 65$ from question 7.
Combine like terms.
Subtract 105 from both sides. |
| <hr/> | | |
| 10. | $180 = y + z$
$180 = 75 + z$
$105 = z$ | Angles y and z are supplemental, since the angle on the lower right of the top triangle is the corresponding angle to y (the vertical angle to the alternate interior angle).
Use $y = 75$ from the previous problem.
Subtract 75 from both sides. |

Explanations for Chapter 9 Practice Drill 3 - Pythagorean Theorem

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| 1. | $a^2 + 12^2 = 20^2$
$AC = 16$ | Use Pythagorean theorem, $a^2 + b^2 = c^2$. Since AB is opposite angle C , it is the hypotenuse.
Or use the fact that this is a 3:4:5 triangle multiplied by 4: $3 \times 4 = 12$ and $5 \times 4 = 20$, so $4 \times 4 = 16$. |
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	$3^2 + 5^2 = c^2$	The hypotenuse is side AB , opposite angle C , so this is <i>not</i> a 3:4:5 triangle. If the hypotenuse were 5, it would be, but we have legs of 3 and 5.
2.	$9 + 25 = c^2$ $34 = c^2$ $\sqrt{34} = c$	Combine like terms. Square root both sides.
3.	$24^2 + 10^2 = c^2$ $AC = 26$	Use Pythagorean theorem, $a^2 + b^2 = c^2$. Since AB (not given) is opposite angle C , it is the hypotenuse, and BC and AC are both legs. Or use the fact that this is a 5:12:13 triangle multiplied by 4: $5 \times \underline{2} = 10$ and $12 \times \underline{2} = 24$, so $13 \times \underline{2} = 26$.
4.	$2^2 + 3^2 = c^2$ $4 + 9 = c^2$ $13 = c^2$ $\sqrt{13} = c$	Use Pythagorean theorem, $a^2 + b^2 = c^2$. Since AB (not given) is opposite angle C , it is the hypotenuse, and BC and AC are both legs. Combine like terms. Take the square root of both sides.
5.	$0.3^2 + b^2 = 0.5^2$ $AC = 0.4$	Use Pythagorean theorem, $a^2 + b^2 = c^2$. Since AB is opposite angle C , it is the hypotenuse. Or use the fact that this is a 3:4:5 triangle multiplied by 0.1: $3 \times \underline{0.1} = 0.3$ and $5 \times \underline{0.1} = 0.5$, so $4 \times \underline{0.1} = 0.4$.
6.	$5^2 + b^2 = 12^2$ $25 + b^2 = 144$ $b^2 = 119$ $b = \sqrt{119}$	Use Pythagorean theorem, $a^2 + b^2 = c^2$. Since AB is opposite angle C , it is the hypotenuse. Notice that this <i>isn't</i> a 5:12:13 triangle, since the hypotenuse isn't 13. Subtract 25 from both sides. Take the square root of both sides.
7.	$8^2 + 9^2 = c^2$ $64 + 81 = c^2$ $145 = c^2$ $\sqrt{145} = c$	Use Pythagorean theorem, $a^2 + b^2 = c^2$. Since AB (not given) is opposite angle C , it is the hypotenuse, and BC and AC are both legs. Combine like terms. Take the square root of both sides.
8.	$2^2 + b^2 = 10^2$ $4 + b^2 = 100$ $b^2 = 96$ $b = \sqrt{96}$	Use Pythagorean theorem, $a^2 + b^2 = c^2$. Since AB is opposite angle C , it is the hypotenuse. Subtract 4 from both sides.

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| | $8^2 + 8^2 = c^2$ | Use Pythagorean theorem, $a^2 + b^2 = c^2$. Since AB (not given) is opposite angle C , it is the hypotenuse, and BC and AC are both legs. |
| 9. | $64 + 64 = c^2$
$128 = c^2$
$\sqrt{128} = c$ | Combine like terms.
Take the square root of both sides. |
| 10. | $(\frac{3}{7})^2 + (\frac{4}{7})^2 = c^2$
$AC = \frac{5}{7}$ | Use Pythagorean theorem, $a^2 + b^2 = c^2$. Since AB is opposite angle C , it is the hypotenuse.
Or use the fact that this is a 3:4:5 triangle multiplied by $\frac{1}{7}$: $3 \times \frac{1}{7} = \frac{3}{7}$ and $4 \times \frac{1}{7} = \frac{4}{7}$, so $5 \times \frac{1}{7} = \frac{5}{7}$ |

Explanations for Chapter 9 Practice Drill 4 - Special Right Triangles

- | | | |
|----|---|--|
| 1. | $x = 3$
$BC = 3\sqrt{2}$ | Use the ratio of sides for a $45^\circ - 45^\circ - 90^\circ$: $x : x : x\sqrt{2}$. Side AB is a leg, so $x = 3$.
Find BC , the hypotenuse. |
| 2. | $7\sqrt{2} = x\sqrt{2}$
$AC = x = 7$ | Use the ratio of sides for a $45^\circ - 45^\circ - 90^\circ$: $x : x : x\sqrt{2}$. Side BC is a hypotenuse, so $x = 7$.
Find AC , one of the legs. |
| 3. | $x = 4$
$EF = 2x = 2(4) = 8$ | Use the ratio of sides for a $30^\circ - 60^\circ - 90^\circ$: $x : x\sqrt{3} : 2x$. Side ED is the leg opposite the 30° , so $x = 4$.
Find EF , the hypotenuse. |
| 4. | $2x = 12$
$DF = x\sqrt{3} = 6\sqrt{3}$ | Use the ratio of sides for a $30^\circ - 60^\circ - 90^\circ$: $x : x\sqrt{3} : 2x$. Side EF is the hypotenuse, so $x = 6$.
Find DF , the leg opposite the 60° . |
| 5. | $x\sqrt{3} = 20\sqrt{3}$
$EF = 2x = 2(20) = 40$ | Use the ratio of sides for a $30^\circ - 60^\circ - 90^\circ$: $x : x\sqrt{3} : 2x$. Side DF is the opposite the 60° , so $x = 20$.
Find EF , the hypotenuse. |
| 6. | $x = 11$
$BC = 11\sqrt{2}$
$AB + AC + BC = 11 + 11 + 11\sqrt{2}$
$22 + 11\sqrt{2}$ | Use the ratio of sides for a $45^\circ - 45^\circ - 90^\circ$: $x : x : x\sqrt{2}$. Side AC is a leg, so $x = 11$.
Find BC , the hypotenuse.
Find the perimeter of ABC by adding up the side lengths.
Combine like terms. |
| 7. | $x + x\sqrt{3} + 2x$
$3x + x\sqrt{3} = 15 + 5\sqrt{3}$
$EF = 2x = 2(5) = 10$ | Use the ratio of sides for a $30^\circ - 60^\circ - 90^\circ$: $x : x\sqrt{3} : 2x$. Find the perimeter by adding up the side lengths.
Compare the ratio to the perimeter given in the question, and $x = 5$.
Find EF , the hypotenuse. |

-
8. $10 = x\sqrt{2}$
 $\frac{10}{\sqrt{2}} = x$
 $AB = x = \frac{10}{\sqrt{2}}$ or $5\sqrt{2}$
- Use the ratio of sides for a $45^\circ - 45^\circ - 90^\circ$: $x : x : x\sqrt{2}$. Side BC is a hypotenuse, $10 = x\sqrt{2}$. Not as nice as we're used to, but we can still find x .
- Divide both sides by $\sqrt{2}$.
- Find AB , one of the legs.
-
9. $2x = 4\sqrt{3}$
 $DF = x\sqrt{3} = 2\sqrt{3}\sqrt{3} = 2(3) = 6$
- Use the ratio of sides for a $30^\circ - 60^\circ - 90^\circ$: $x : x\sqrt{3} : 2x$. Side EF is the hypotenuse, so $x = 2\sqrt{3}$.
- Find DF , the leg opposite the 60° . Remember that $\sqrt{x}^2 = x$.
-
10. $x\sqrt{3} = \sqrt{6}$
 $x\sqrt{3} = \sqrt{2}(\sqrt{3})$
 $EF = 2x = 2\sqrt{2}$
- Use the ratio of sides for a $30^\circ - 60^\circ - 90^\circ$: $x : x\sqrt{3} : 2x$. Side DF is the opposite the 60° , so $x\sqrt{3} = \sqrt{6}$.
- We could square both sides and solve for x that way, or we can rewrite the right side to make it easier to compare: since $6 = 2(3)$, $\sqrt{6} = \sqrt{2}(\sqrt{3})$ and $x = \sqrt{2}$.
- Find EF , the hypotenuse.
-

Explanations for Chapter 9 Practice Drill 5 - Trigonometry

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1. $\sin B = \frac{15}{17}$
- Use *SOHCAHTOA*: $\sin B = \frac{Opp}{Hyp}$
-
2. $\tan A = \frac{8}{15}$
- Use *SOHCAHTOA*: $\tan A = \frac{Opp}{Adj}$
-
3. $\cos B = \frac{8}{17}$
- Use *SOHCAHTOA*: $\cos B = \frac{Adj}{Hyp}$
-
4. $\cos E = \frac{2}{4} = \frac{1}{2}$
- Use *SOHCAHTOA*: $\cos E = \frac{Adj}{Hyp}$
-
5. $2^2 + b^2 = c^2$
 $\cos D = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$
- Use *SOHCAHTOA*: for cosine, we need to know the side adjacent to D , which we can find with Pythagorean theorem. Or, you may notice that DEF is a $30^\circ - 60^\circ - 90^\circ$, and the ratios of sides are $x : x\sqrt{3} : 2x$. That means that $x = 2$ and $DF = 2\sqrt{3}$.
- $\cos D = \frac{Adj}{Hyp}$
-
6. $\tan E = \frac{2\sqrt{3}}{2} = \sqrt{3}$
- Use *SOHCAHTOA*: $\tan E = \frac{Opp}{Adj}$, and the length of DF is $2\sqrt{3}$ from question 5.
-

-
7. $12^2 + b^2 = 14^2$ Use *SOHCAHTOA*: for sine, we need to know the side opposite to R , which we can find with Pythagorean theorem.
- $144 + b^2 = 196$
- $b^2 = 52$ Subtract 144 from both sides.
- $b = \sqrt{52}$ Take the square root of both sides.
- $\sin R = \frac{\sqrt{52}}{12}$ or $\frac{\sqrt{13}}{6}$ $\sin R = \frac{Opp}{Hyp}$
-
8. $\cos R = \frac{12}{13} = \frac{12}{x}$ Use *SOHCAHTOA*, so $\cos R = \frac{Adj}{Hyp}$, and the value of cosine given in the problem. Since the adjacent side is 12, the hypotenuse is 13, and we have a 5:12:13 triangle, and $ST = 5$.
- $\tan R = \frac{5}{12}$ Use *SOHCAHTOA*: $\tan A = \frac{Opp}{Adj}$
-
9. $\tan R = \frac{3}{4} = \frac{x}{12}$ Use *SOHCAHTOA*: $\tan R = \frac{Opp}{Adj}$, and the value of tangent given in the problem. Solve for x by cross multiplying or finding the multiplier, and you'll get x , the opposite side ST , is 9. (This is a 3:4:5 triangle times 3, by the way.)
-
10. $\cos S = \frac{4}{5} = \frac{ST}{RS}$ Use *SOHCAHTOA*, so $\cos S = \frac{Adj}{Hyp}$. We don't know the length of ST or RS , but we now know the ratio of the sides, and this is a 3:4:5 triangle. $ST = 4x$, $RS = 5x$, and $RT = 3x = 12$.
- $12 : 16 : 20$ Find all 3 sides of the triangle. $RS = 20$.

1.9 Chapter 10 Quadratics Answers

Explanations for Chapter 10 Practice Drill 1 - Factoring

-
- | | | |
|-------|--|--|
| 1. | $(x+7)(x+3) = 0$
$x = -7$ or $x = -3$ | Find two numbers that Add to 10 and Multiply to 21: 3 and 7.
Either $(x+7) = 0$ or $(x+3) = 0$. |
| <hr/> | | |
| 2. | $(x+4)(x-6) = 0$
$x = -4$ or $x = 6$ | Find two numbers that Add to -2 and Multiply to -24 : 4 and -6 .
Either $(x+4) = 0$ or $(x-6) = 0$. |
| <hr/> | | |
| 3. | $(x-1)(x+2) = 0$
$x = 1$ or $x = -2$ | Find two numbers that Add to 1 and Multiply to -2 : -1 and 2.
Either $(x-1) = 0$ or $(x+2) = 0$. |
| <hr/> | | |
| 4. | $(x-3)(x-8) = 0$
$x = 3$ or $x = 8$ | Find two numbers that Add to -11 and Multiply to 24: -3 and -7 .
Either $(x-3) = 0$ or $(x-8) = 0$. |
| <hr/> | | |
| 5. | $(x-5)(x+12) = 0$
$x = 5$ or $x = -12$ | Find two numbers that Add to 7 and Multiply to -60 : -5 and 12.
Either $(x-5) = 0$ or $(x+12) = 0$. |
| <hr/> | | |
| 6. | $(x-1)(x-3) = 0$
$x = 1$ or $x = 3$ | Find two numbers that Add to -4 and Multiply to 3: -1 and -3 .
Either $(x-1) = 0$ or $(x-3) = 0$. |
| <hr/> | | |
| 7. | $(x-1)(x+5) = 0$
$x = 1$ or $x = -5$ | Find two numbers that Add to 4 and Multiply to -5 : -1 and 5.
Either $(x-1) = 0$ or $(x+5) = 0$. |
| <hr/> | | |
| 8. | $(x+1)^2 = 0$
$x = -1$ | Find two numbers that Add to 2 and Multiply to 1: 1 and 1.
$(x+1) = 0$. |
| <hr/> | | |
| 9. | $x(x+4) = 0$
$x = 0$ or $x = -4$ | We could find two numbers that Add to 4 and Multiply to 0, but there's another way to think of this factoring: what do both terms have in common? Since they both have an x , factor out an x .
Either $x = 0$ or $(x+4) = 0$. |
| <hr/> | | |
| 10. | $(x+7)(x+11) = 0$
$x = -7$ or $x = -11$ | Find two numbers that Add to 18 and Multiply to 77: 7 and 11.
Either $(x+7) = 0$ or $(x+11) = 0$. |

Explanations for Chapter 10 Practice Drill 2 - Factoring More Difficult Quadratics

-
- | | | |
|----|--|--|
| 1. | $x^2 + 11x + 18 = 0$
$(x+2)(x+9) = 0$
$x = -2$ or $x = -9$ | Add 18 to both sides to set the equation equal to zero.
Find two numbers that Add to 11 and Multiply to 18: 2 and 9.
Either $(x+2) = 0$ or $(x+9) = 0$ |
|----|--|--|

2. $x^2 + x - 12 = 0$ Distribute the x and subtract 12 from both sides to set the equation equal to zero.
 $(x - 3)(x + 4) = 0$ Find two numbers that Add to 1 and Multiply to -12 : -3 and 4 .
 $x = 3$ or $x = -4$ Either $(x - 3) = 0$ or $(x + 4) = 0$

3. $x^2 - x - 30$ Subtract x from both sides to set the equation equal to zero.
 $(x + 5)(x - 6) = 0$ Find two numbers that Add to -1 and Multiply to -30 : 5 and -6 .
 $x = -5$ or $x = 6$ Either $(x + 5) = 0$ or $(x - 6) = 0$.

4. $x^2 + 5x - 14 = 0$ Subtract $x + 14$ from both sides to set the equation equal to zero.
 $(x - 2)(x + 7) = 0$ Find two numbers that Add to 5 and Multiply to -14 : -2 and 7 .
 $x = 2$ or $x = -7$ Either $(x - 2) = 0$ or $(x + 7) = 0$.

5. $x^2 - 3x - 18 = 0$ Since all the terms are divisible by 2 , divide the entire equation by 2 .
 $(x + 3)(x - 6) = 0$ Find two numbers that Add to -3 and Multiply to -18 : 3 and -6 .
 $x = -3$ or $x = 6$ Either $(x + 3) = 0$ or $(x - 6) = 0$.

6. $x^2 - x - 56 = 0$ Subtract x and add 14 from both sides to set the equation equal to zero.
 $(x + 7)(x - 8) = 0$ Find two numbers that Add to -1 and Multiply to -56 : 7 and -8 .
 $x = -7$ or $x = 8$ Either $(x + 7) = 0$ or $(x - 8) = 0$.

7. $x^2 - 7x + 6 = 0$ Since all the terms are divisible by 5 , divide the entire equation by 5 .
 $(x - 1)(x - 6) = 0$ Find two numbers that Add to -7 and Multiply to 6 : -1 and -6 .
 $x = 1$ or $x = 6$ Either $(x - 1) = 0$ or $(x - 6) = 0$.

8. $x^2 + 13x - 48 = 0$ Subtract $x^2 + 2x + 18$ from both sides to set the equation equal to 0 .
 $(x - 3)(x + 16) = 0$ Find two numbers that Add to 13 and Multiply to -48 : -3 and 16 .
 $x = 3$ or $x = -16$ Either $(x - 3) = 0$ or $(x + 16) = 0$.

9. $x(x - 5) = 6(x - 4)$ Since we have a fraction equals a fraction, cross multiply.
 $x^2 - 5x = 6x - 24$ Distribute.
 $x^2 - 11x + 24 = 0$ Subtract $6x$ and add 24 to both sides to set the equation equal to zero.
 $(x - 3)(x - 8) = 0$ Find two numbers that Add to -11 and Multiply to 24 : -3 and -8 .
 $x = 3$ or $x = 8$ Either $(x - 3) = 0$ or $(x - 8) = 0$.

10. $x(x) + 1(x+2) = -2(x)$
 $x^2 + x + 2 = -2x$
 $x^2 + 3x + 2 = 0$
 $(x+1)(x+2) = 0$
 $x = -1$ or $x = -2$
 $\frac{-2}{-2+2} \dots$
- Get rid of all the denominators. Multiply the entire equation by $x(x+2)$. In the first and last terms, the $x+2$ will cancel out, and in the middle term the x term will cancel out.
- Simplify.
- Add $2x$ to both sides to set the equation equal to zero.
- Find two numbers that Add to 3 and Multiply to 2: 1 and 2.
- Either $(x+1) = 0$ or $(x+2) = 0$.
- Try out both answers in the denominators from the original equation, to make sure you don't end with a denominator of 0. $x = -1$ works, but $x = -2$ will give you zeroes in the denominators, so it is bad. It's just a no good answer, get rid of it.

Explanations for Chapter 10 Practice Drill 3 - The Quadratic Formula

1. $x = \frac{-7 \pm \sqrt{7^2 - 4(2)(5)}}{2(2)}$
 $x = \frac{-7 \pm \sqrt{49 - 40}}{4}$
 $x = \frac{-7 \pm \sqrt{9}}{4}$
 $x = \frac{-7 \pm 3}{4}$
 $x = \frac{-7+3}{4}$, or $x = \frac{-7-3}{4}$
 $x = -1$ or $\frac{-5}{2}$
- Use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, with $a = 2$, $b = 7$, and $c = 5$.
- Simplify.
- Separate out the \pm
- Simplify each fraction.
-
2. $3x^2 + x - 1 = 0$
 $x = \frac{-1 \pm \sqrt{1^2 - 4(3)(-1)}}{2(3)}$
 $x = \frac{-1 \pm \sqrt{1+12}}{6}$
 $x = \frac{-1 \pm \sqrt{13}}{6}$
- Add x and subtract 1 from both sides to set the equation equal to zero.
- Use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, with $a = 3$, $b = 1$, and $c = -1$.
- Simplify.
-
3. $x = \frac{-12 \pm \sqrt{12^2 - 4(4)(9)}}{2(4)}$
 $x = \frac{-12 \pm \sqrt{144 - 144}}{8}$
 $x = \frac{-12 \pm \sqrt{0}}{8}$
 $x = \frac{-12}{8} = -\frac{3}{2}$
- Use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, with $a = 4$, $b = 12$, and $c = 9$.
- Simplify.
- Simplify with $\sqrt{0} = 0$, then simplify the fraction.
-
4. $x^2 + 5x - 11 = 0$
 $x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-11)}}{2(1)}$
 $x = \frac{-5 \pm \sqrt{25+44}}{2}$
 $x = \frac{-5 \pm \sqrt{69}}{2}$
- Subtract 11 from both sides to set the equation equal to zero.
- Use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, with $a = 1$, $b = 5$, and $c = -11$.
- Simplify.

1.9 Chapter 10 Quadratics Answers and Explanations

5.
$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-12)}}{2(2)}$$

Use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, with $a = 2$, $b = 4$, and $c = -12$.

$$x = \frac{-4 \pm \sqrt{16 + 96}}{4}$$

Simplify.

$$x = \frac{-4 \pm \sqrt{112}}{4}$$

Separate out $112 = 16 \times 7$.

$$x = \frac{-4 \pm \sqrt{16}\sqrt{7}}{4}$$

$$x = \frac{-4 \pm 4\sqrt{7}}{4}$$

$$x = -1 \pm 1\sqrt{7}$$

Factor out 4 to simplify the fraction.

6.
$$x = \frac{-1 \pm \sqrt{1^2 - 4(-1)(8)}}{2(-1)}$$

Use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, with $a = -1$, $b = 1$, and $c = 8$.

$$x = \frac{-1 \pm \sqrt{1 + 32}}{-2}$$

Simplify.

$$x = \frac{1 \pm \sqrt{33}}{2}$$

Cancel out the negatives.

7.
$$x = \frac{-12 \pm \sqrt{12^2 - 4(6)(3)}}{2(6)}$$

Use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, with $a = 6$, $b = 12$, and $c = 3$.

$$x = \frac{-12 \pm \sqrt{144 - 72}}{12}$$

Simplify.

$$x = \frac{-12 \pm \sqrt{72}}{12}$$

Break down $\sqrt{72}$ as $\sqrt{36}\sqrt{2}$.

$$x = \frac{-12 \pm \sqrt{36}\sqrt{2}}{12}$$

$$x = \frac{-12 \pm 6\sqrt{2}}{12}$$

$$x = \frac{6(-2 \pm \sqrt{2})}{12}$$

Factor out a 6 from both terms on top.

$$x = \frac{-2 \pm \sqrt{2}}{2}$$

Reduce the fraction.

8.
$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(0.5)}}{2(1)}$$

Use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, with $a = 1$, $b = 3$, and $c = 0.5$.

$$x = \frac{-3 \pm \sqrt{9 - 2}}{2}$$

Simplify.

$$x = \frac{-3 \pm \sqrt{7}}{2}$$

9.
$$x^2 + 3x = -2x - 2$$

Distribute in each parentheses.

$$x^2 + 5x + 2 = 0$$

Add $2x + 2$ to both sides to set the equation equal to zero.

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(2)}}{2(1)}$$

Use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, with $a = 1$, $b = 5$, and $c = 2$.

$$x = \frac{-5 \pm \sqrt{25 - 8}}{2}$$

Simplify.

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

10. $2x^2 + 6x + 1 = 0$
 $x = \frac{-6 \pm \sqrt{6^2 - 4(2)(1)}}{2(2)}$
 $x = \frac{-6 \pm \sqrt{36 - 8}}{4}$
 $x = \frac{-6 \pm \sqrt{28}}{4}$
 $x = \frac{-6 \pm \sqrt{4}\sqrt{7}}{4}$
 $x = \frac{-6 \pm 2\sqrt{7}}{4}$
 $x = \frac{2(-3 \pm \sqrt{7})}{4}$
 $x = \frac{-3 \pm \sqrt{7}}{2}$
- Subtract everything on the right side of the equation to set it all equal to zero.
 Use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, with $a = 2$, $b = 6$, and $c = 1$.
 Simplify.
 Break apart $\sqrt{28} = \sqrt{4}\sqrt{7}$
 $\sqrt{4} = 2$.
 Factor out a 2 on the top.
 Reduce the fraction.

Explanations for Chapter 10 Practice Drill 4 - The Discriminant

1. $7^2 - 4(2)(-3)$
 $49 - (-24)$
 $73 > 0$
- Use $b^2 - 4ac$ with $a = 2$, $b = 7$, and $c = -3$.
 Simplify.
 Since the discriminant is greater than 0, the quadratic has 2 real solutions.

2. $1^2 - 4(5)(1)$
 $1 - (20)$
 $-19 < 0$
- Use $b^2 - 4ac$ with $a = 5$, $b = 1$, and $c = 1$.
 Simplify.
 Since the discriminant is less than 0, the quadratic has no real solutions.

3. $x^2 + 2x + 9 = 0$
 $2^2 - 4(1)(9)$
 $4 - (36)$
 $-32 < 0$
- Add 9 to both sides to set the equation equal to zero.
 Use $b^2 - 4ac$ with $a = 1$, $b = 2$, and $c = 9$
 Simplify.
 Since the discriminant is less than 0, the quadratic has no real solutions.

4. $3x^2 - 30x + 75 = 0$
 $(-30)^2 - 4(3)(75)$
 $900 - (900)$
 $0 = 0$
- Subtract $30x$ from both sides to set the equation equal to zero.
 Use $b^2 - 4ac$ with $a = 3$, $b = -30$, and $c = 75$.
 Simplify.
 Since the discriminant is equal to 0, the quadratic has 1 real solutions.

5. $4x^2 + 12x - 3 = 0$
 $12^2 - 4(4)(-3)$
 $144 - (-48)$
 $192 > 0$
- Distribute on the left side, then subtract 3 from both sides to set the equation equal to zero.
 Use $b^2 - 4ac$ with $a = 4$, $b = 12$, and $c = -3$.
 Simplify.
 Since the discriminant is greater than 0, the quadratic has 2 real solutions.

1.9 Chapter 10 Quadratics Answers and Explanations

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|-------|-----------------------------------|--|
| | $8^2 - 4a(2) = 0$ | The equation should have exactly one solution, so set the discriminant, $b^2 - 4ac$, equal to zero, with $b = 8$ and $c = 2$. |
| 6. | $64 - 8a = 0$ | Simplify. |
| | $64 = 8a$ | Add $8a$ to both sides. |
| | $8 = a$ | Divide both sides by 8. |
| <hr/> | | |
| | $b^2 - 4(1)(9) = 0$ | The equation should have exactly one solution, so set the discriminant, $b^2 - 4ac$, equal to zero, with $a = 1$ and $c = 9$. |
| 7. | $b^2 - 36 = 0$ | Simplify. |
| | $b^2 = 36$ | Add 36 to both sides. |
| | $b = 6$ or $b = -6$ | Take the square root of both sides. $\sqrt{x^2} = \pm x$. |
| <hr/> | | |
| | $b^2 - 4(3)(75) = 0$ | The equation should have exactly one solution, so set the discriminant, $b^2 - 4ac$, equal to zero, with $a = 3$ and $c = 75$. |
| 8. | $b^2 - 900 = 0$ | Simplify. |
| | $b^2 = 900$ | Add 900 to both sides. |
| | $b = 30$ or $b = -30$ | Take the square root of both sides. $\sqrt{x^2} = \pm x$. |
| <hr/> | | |
| | $4^2 - 4(3)c < 0$ | The equation should have no real solutions, so the discriminant, $b^2 - 4ac$, must be less than zero. Use $a = 3$ and $b = 4$. |
| 9. | $16 - 12c < 0$ | Simplify. |
| | $16 < 12c$ | Add $12c$ to both sides. |
| | $\frac{16}{12} = \frac{4}{3} < c$ | Divide both sides by 12 and reduce. Any answer greater than $\frac{4}{3}$ is correct. |
| <hr/> | | |
| | $b^2 - 4(7)(28) = 0$ | The equation should have exactly one solution, so set the discriminant, $b^2 - 4ac$, equal to zero, with $a = 7$ and $c = 28$. |
| 10. | $b^2 - 784 = 0$ | Simplify. |
| | $b^2 = 784$ | Add 784 to both sides. |
| | $b = 28$ or $b = -28$ | Take the square root of both sides. $\sqrt{x^2} = \pm x$ |

Explanations for Chapter 10 Practice Drill 5 - Sum of Solutions

- | | | |
|-------|-----------------------|--|
| 1. | $\frac{-17}{1} = -17$ | The sum of solutions is $\frac{-b}{a}$. |
| <hr/> | | |
| 2. | $\frac{-1}{3}$ | The sum of solutions is $\frac{-b}{a}$. |
| <hr/> | | |
| 3. | $\frac{-7}{1} = -7$ | The sum of solutions is $\frac{-b}{a}$. |
| <hr/> | | |
| 4. | $\frac{-2}{2} = -1$ | The sum of solutions is $\frac{-b}{a}$. |

5. $\frac{-1}{-4} = \frac{1}{4}$ The sum of solutions is $\frac{-b}{a}$.

Explanations for Chapter 10 Practice Drill 6 - Factored Form

1. $(-3, 0)$ and $(-6, 0)$ To find the x -intercepts, either set $y = 0$ and solve, or use the factored form: $y = a(x-r)(x-s)$.
2. $(4, 0)$ and $(-2, 0)$ To find the zeroes, either set $y = 0$ and solve, or use the factored form: $y = a(x-r)(x-s)$.
3. $(-1, 0)$ and $(2, 0)$ To find the x -intercepts, either set $y = 0$ and solve, or use the factored form: $y = a(x-r)(x-s)$.
4. $(-7, 0)$ and $(1, 0)$ Either set $y = 0$ and solve, or use the factored form: $y = a(x-r)(x-s)$.
 $m = 7$ Since $m > n$, $m = 7$.
5. $(3, 0)$ and $(-13, 0)$ To find the zeroes, either set $y = 0$ and solve, or use the factored form: $y = a(x-r)(x-s)$.
6. $y = (x-3)(x-5)$ Use the factored form: $y = a(x-r)(x-s)$, where $a = 1$, $r = 3$ (one of the x -intercepts), and $s = 5$ (the other x -intercept).
7. $y = (x+2)(x-7)$ Use the factored form: $y = a(x-r)(x-s)$, where $a = 1$, $r = -2$ (one of the zeroes), and $s = 7$ (the other zero).
8. $y = (x-10)(x+1)$ Use the factored form: $y = a(x-r)(x-s)$, where $a = 1$, $r = 10$, from the point $(10, 0)$, and $s = -1$, from the point $(-1, 0)$.
9. $y = (x-8)^2$ Use the factored form: $y = a(x-r)(x-s)$, where $a = 1$, $r = 8$, the only x -intercept. This is called a *double root*, because the factor is doubled: $(x-8)(x-8) = (x-8)^2$.
10. $y = (x-8)(x-2)$ We're given two points: $f(8) = 0$ is the point $(8, 0)$, since $f(x) = y$. The other point, given by $f(2) = 0$, is $(2, 0)$. Use the factored form: $y = a(x-r)(x-s)$, where $a = 1$, $r = 8$, from the point $(8, 0)$, and $s = 2$, from the point $(2, 0)$.

Explanations for Chapter 10 Practice Drill 7 - Vertex Form

1. Vertex: $(2, 3)$ Find the vertex (h, k) from the vertex form, $y = a(x-h)^2 + k$, to find that $h = 2$ and $k = 3$.
2. Vertex: $(-7, -1)$ Find the vertex (h, k) from the vertex form, $y = a(x-h)^2 + k$, to find that $h = -7$ and $k = -1$. Notice that, since we have $(x+7)$, the x -coordinate of the vertex is *negative 7*.

1.9 Chapter 10 Quadratics Answers and Explanations

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|-----|---------------------|---|
| 3. | Vertex: $(-1, 5)$ | Find the vertex (h, k) from the vertex form, $y = a(x - h)^2 + k$, to find that $h = -1$ and $k = 5$. |
| 4. | Vertex: $(0, 2)$ | Find the vertex (h, k) from the vertex form, $y = a(x - h)^2 + k$, to find that $h = 0$ and $k = 2$. |
| 5. | Vertex: $(-4, 0)$ | Find the vertex (h, k) from the vertex form, $y = a(x - h)^2 + k$, to find that $h = -4$ and $k = 0$. |
| 6. | $y = (x - 3)^2 + 5$ | Use $a = 1$ and the vertex $(3, 5)$ in the vertex form, $y = a(x - h)^2 + k$. |
| 7. | $y = (x + 5)^2 + 1$ | Use $a = 1$ and the vertex $(-5, 1)$ in the vertex form, $y = a(x - h)^2 + k$. |
| 8. | $y = x^2 + 11$ | Use $a = 1$ and the vertex $(0, 11)$ in the vertex form, $y = a(x - h)^2 + k$. |
| 9. | $y = (x - 1)^2 - 8$ | Use $a = 1$ and the vertex $(1, -8)$ in the vertex form, $y = a(x - h)^2 + k$. |
| 10. | $y = x^2$ | Use $a = 1$ and the vertex $(0, 0)$ in the vertex form, $y = a(x - h)^2 + k$. |

Explanations for Chapter 10 Practice Drill 8 - Constants and Coefficients

- | | | |
|----|-------------|--|
| 1. | D only | The equation is in factored form, $y = a(x - r)(x - s)$, which gives us the x -coordinates of the x -intercepts, $(r, 0)$ and $(s, 0)$. |
| 2. | A, B, and E | The equation is in vertex form, $y = a(x - h)^2 + k$, which gives us the x - and y -coordinates of the vertex at (h, k) . The x -intercept of the line of symmetry is also the x -coordinate of the vertex. |
| 3. | D only | The equation is in factored form, $y = a(x - r)(x - s)$, which gives us the x -coordinates of the x -intercepts, $(r, 0)$ and $(s, 0)$. |
| 4. | C only | The equation is in standard form, $y = ax^2 + bx + c$, which gives us the y -intercept at $(0, c)$. |
| 5. | A, B, and E | The equation is in vertex form, $y = a(x - h)^2 + k$, which gives us the x - and y -coordinates of the vertex at (h, k) . The x -intercept of the line of symmetry is also the x -coordinate of the vertex. |
| 6. | D only | The equation is in factored form, $y = a(x - r)(x - s)$, which gives us the x -coordinates of the x -intercepts, $(r, 0)$ and $(s, 0)$. |
| 7. | C only | The equation is in standard form, $y = ax^2 + bx + c$, which gives us the y -intercept at $(0, c)$. |

-
8. D only
The equation is in factored form, $y = a(x - r)(x - s)$, which gives us the x -coordinates of the x -intercepts, $(r, 0)$ and $(s, 0)$. In this case, $r = 0$ and $s = -1$.
-
9. A, B, D, and E
The equation is in vertex form, $y = a(x - h)^2 + k$, which gives us the x - and y -coordinates of the vertex at (h, k) . The x -intercept of the line of symmetry is also the x -coordinate of the vertex. Since the vertex, $(5, 0)$, is also the only x -intercept, this equation is also in factored form, $y = a(x - r)(x - s)$, with a double root at $x = 5$, which gives us the x -coordinates of the x -intercept at $(5, 0)$.
-
10. A, B, C, and E
The equation is in vertex form, $y = a(x - h)^2 + k$, which gives us the x - and y -coordinates of the vertex at (h, k) . The x -intercept of the line of symmetry is also the x -coordinate of the vertex. The vertex is also the y -intercept, $(0, 6)$, and this equation is also in standard form, $y = ax^2 + bx + c$, where $b = 0$.

Explanations for Chapter 10 Practice Drill 9 - Identify the Graph

-
1. G
The equation is in factored form, $y = a(x - r)(x - s)$, which gives us the x -intercepts, $(r, 0)$ and $(s, 0)$. Here, $a = 0.5$, $r = -2$, and $s = 3$, so the zeroes are $(-2, 0)$ and $(3, 0)$ and the parabola opens up. The only graph with those x -intercepts is (G).
-
2. F
The equation is in factored form, $y = a(x - r)(x - s)$, which gives us the x -intercepts, $(r, 0)$ and $(s, 0)$. Here, $a = -1$, $r = -1$, and $s = -4$, so the zeroes are $(-1, 0)$ and $(-4, 0)$ and the parabola opens down. The only graph with those x -intercepts is (F).
-
3. I
The equation is in vertex form, $y = a(x - h)^2 + k$, which gives us the coordinates of the vertex, (h, k) . Here, $a = -2$, $h = -3$, and $k = -3$, so the vertex is at $(-3, -3)$ and the parabola opens down. The only graph with that vertex is (I).
-
4. A
The equation is in vertex form, $y = a(x - h)^2 + k$, which gives us the coordinates of the vertex, (h, k) . Here, $a = 1$, $h = 3$, and $k = 1$, so the vertex is at $(3, 1)$ and the parabola opens up. The only graph with that vertex is (A).
-
5. C
The equation is in factored form, $y = a(x - r)(x - s)$, which gives us the x -intercepts, $(r, 0)$ and $(s, 0)$. Here, $a = 1$, $r = 0$, and $s = 2$, so the zeroes are $(0, 0)$ and $(2, 0)$ and the parabola opens up. The only graph with those x -intercepts is (C).
-
6. J
The equation is in vertex form, $y = a(x - h)^2 + k$, which gives us the coordinates of the vertex, (h, k) . Here, $a = -1$, $h = -1$, and $k = 2$, so the vertex is at $(-1, 2)$ and the parabola opens down. The only graph with that vertex is (J).
-
7. H
The equation is in vertex form, $y = a(x - h)^2 + k$, which gives us the coordinates of the vertex, (h, k) . Here, $a = -1$, $h = 4$, and $k = 0$, so the vertex is at $(4, 0)$ and the parabola opens down. The only graph with that vertex is (H). You could also think of this equation as factored form, $y = a(x - r)(x - s)$, where $r = s = 4$, and the only zero is the double root $(4, 0)$.
-
8. L
The equation is in factored form, $y = a(x - r)(x - s)$, which gives us the x -intercepts, $(r, 0)$ and $(s, 0)$. Here, $a = -0.25$, $r = -4$, and $s = 4$, so the zeroes are $(-4, 0)$ and $(4, 0)$ and the parabola opens down. The only graph with those x -intercepts is (L).

9. D The equation is in vertex form, $y = a(x - h)^2 + k$, which gives us the coordinates of the vertex, (h, k) . Here, $a = -2$, $h = 3$, and $k = -3$, so the vertex is at $(3, -3)$ and the parabola opens down. The only graph with that vertex is (D).

10. E The equation is in vertex form, $y = a(x - h)^2 + k$, which gives us the coordinates of the vertex, (h, k) . Here, $a = 1$, $h = -2$, and $k = -1$, so the vertex is at $(-2, -1)$ and the parabola opens up. The only graph with that vertex is (E).

Explanations for Chapter 10 Practice Drill 10 - Substitution

1. $m = x^2$ This is similar to a quadratic, but our b term is x^2 instead of x , so set $m = x^2$.
 $m^2 + 6m + 9$ Rewrite the equation.
 $(m + 3)^2$ Find a pair of numbers that Add to 6 and Multiply to 9: 3 and 3.
 $(x^2 + 3)^2$ Replace $m = x^2$.

2. $m = x^2$ This is similar to a quadratic, but our b term is x^2 instead of x , so set $m = x^2$.
 $m^2 - 8m + 16$ Rewrite the equation.
 $(m - 4)^2$ Find a pair of numbers that Add to -8 and Multiply to 16: -4 and -4 .
 $(x^2 - 4)^2$ Replace $m = x^2$
 $(x - 2)^2(x + 2)^2$ $x^2 - 4$ is the difference of squares: $x^2 - y^2 = (x - y)(x + y)$, and can be rewritten as $(x - 2)(x + 2)$. Since we have $(x^2 - 4)^2 = (x^2 - 4)(x^2 - 4) = (x - 2)(x + 2)(x - 2)(x + 2)$

3. $m = x^2$ This is similar to a quadratic, but our b term is x^2 instead of x , so set $m = x^2$.
 $m^2 + 10m + 25$ Rewrite the equation.
 $(m + 5)^2$ Find a pair of numbers that Add to 10 and Multiply to 25: 5 and 5.
 $(x^2 + 5)^2$ Replace $m = x^2$

4. $(a^2 - b^2)(a^2 + b^2)$ $a^4 - b^4$ is the difference of squares: $x^2 - y^2 = (x - y)(x + y)$, where $x = a^2$ and $y = b^2$.
 $(a - b)(a + b)(a^2 + b^2)$ Now $(a^2 - b^2)$ is the difference of squares, so rewrite it as $(a - b)(a + b)$.

5. $x^4 - 10x^2 + 9 = 0$ Subtract $10x^2$ from both sides to set the equation equal to zero.
 $m = x^2$ This is similar to a quadratic, but our b term is x^2 instead of x , so set $m = x^2$.
 $m^2 - 10m + 9 = 0$ Find a pair of numbers that Add to -10 and Multiply to 9: -1 and -9 .
 $(m - 1)(m - 9) = 0$ Factor.
 $(x^2 - 1)(x^2 - 9) = 0$ Replace $m = x^2$
 $(x - 1)(x + 1)(x - 3)(x + 3) = 0$ Both $(x^2 - 1)$ and $(x^2 - 9)$ are the difference of squares: $x^2 - y^2 = (x - y)(x + y)$. For $(x^2 - 9)$, we have x^2 and y^2 is $9 = 3^2$. The first factor, $(x^2 - 1)$, is less obvious: we have x^2 and y^2 is 1, which is the same as 1^2 .
 $x = -1, 1, -3, \text{ or } 3$ Set each factor equal to zero and solve.
 $x = 1 \text{ or } 3$ Since the drill asks for solutions where $x > 0$, ignore the negative solutions.

6. $m = x^2$
 $m^2 - 29m + 100 = 0$
 $(m - 4)(m - 25) = 0$
 $(x^2 - 4)(x^2 - 25) = 0$
 $(x - 2)(x + 2)(x - 5)(x + 5) = 0$
 $x = -2, 2, -5, \text{ or } 5.$
 $x = 2 \text{ or } 5$
- This is similar to a quadratic, but our b term is x^2 instead of x , so set $m = x^2$.
 Find a pair of numbers that Add to -29 and Multiply to 100 : -4 and -25 .
 Factor.
 Replace $m = x^2$
 Both $(x^2 - 4)$ and $(x^2 - 9)$ are the difference of squares: $x^2 - y^2 = (x - y)(x + y)$.
 For $(x^2 - 4)$, we have x^2 and $y^2 = 4 = 2^2$, and for $(x^2 - 9)$, we have x^2 and $y^2 = 9 = 3^2$.
 Set each factor equal to zero and solve.
 Since the drill asks for solutions where $x > 0$, ignore the negative solutions.

7. $x^4 - 5x^2 + 4 = 0$
 $m = x^2$
 $m^2 - 5m + 4 = 0$
 $(m - 1)(m - 4) = 0$
 $(x^2 - 1)(x^2 - 4) = 0$
 $(x - 1)(x + 1)(x - 2)(x + 2) = 0$
 $x = -1, 1, -2, \text{ or } 2.$
 $x = 1 \text{ or } 2$
- Divide the entire equation by x . (Technically this gets rid of a possible solution for the equation, of $x = 0$. Since the drill says that $x > 0$, we can ignore that solution.)
 This is similar to a quadratic, but our b term is x^2 instead of x , so set $m = x^2$.
 Find a pair of numbers that Add to -5 and Multiply to 4 : -1 and -4 .
 Factor.
 Replace $m = x^2$
 Both $(x^2 - 1)$ and $(x^2 - 4)$ are the difference of squares: $x^2 - y^2 = (x - y)(x + y)$.
 For $(x^2 - 9)$, we have x^2 and $y^2 = 4 = 2^2$. The first factor, $(x^2 - 1)$, is less obvious: we have x^2 and $y^2 = 1 = 1^2$.
 Set each factor equal to zero and solve.
 Since the drill asks for solutions where $x > 0$, ignore the negative solutions.

8. $x^4 - 17x^2 + 16$
 $m = x^2$
 $m^2 - 17m + 16 = 0$
 $(m - 1)(m - 16) = 0$
 $(x^2 - 1)(x^2 - 16) = 0$
 $(x - 1)(x + 1)(x - 4)(x + 4) = 0$
 $x = -1, 1, -4, \text{ or } 4.$
 $x = 1 \text{ or } 4$
- Subtract $7x^2$ from both sides and add 16 to both sides to set the equation equal to zero.
 This is similar to a quadratic, but our b term is x^2 instead of x , so set $m = x^2$.
 Find a pair of numbers that Add to -17 and Multiply to 16 : -1 and -16 .
 Factor.
 Replace $m = x^2$
 Both $(x^2 - 1)$ and $(x^2 - 16)$ are the difference of squares: $x^2 - y^2 = (x - y)(x + y)$.
 For $(x^2 - 16)$, we have x^2 and $y^2 = 16 = 4^2$. So $(x^2 - 16) = (x^2 - 4^2)$.
 The first factor, $(x^2 - 1)$, is less obvious: we have x^2 and $y^2 = 1 = 1^2$, so $(x^2 - 1) = (x^2 - 1^2)$.
 Set each factor equal to zero and solve.
 Since the drill asks for solutions where $x > 0$, ignore the negative solutions.

- 9.
- | | |
|--|--|
| $m = x^2$ | This is similar to a quadratic, but our b term is \sqrt{x} instead of x , so set $m = \sqrt{x}$. That also means $m^2 = (\sqrt{x})^2 = x$. |
| $m^2 - 4m - 5 = 0$ | Find a pair of numbers that Add to -4 and Multiply to -5 : 1 and -5 . |
| $(m + 1)(m - 5) = 0$ | Factor. |
| $(\sqrt{x} + 1)(\sqrt{x} - 5) = 0$ | Replace $m = \sqrt{x}$. |
| $(\sqrt{x} + 1) = 0$ or $(\sqrt{x} - 5) = 0$ | Set each factor equal to zero and solve. |
| $\sqrt{x} = -1$ or $\sqrt{x} = 5$ | Since a square root can never give us a negative answer, we can ignore the first equation and just focus on $\sqrt{x} = 5$. |
| $x = 25$ | Square both sides of the equation to get rid of the square root. |
-

- 10.
- | | |
|------------------------------------|---|
| $m = x^3$ | This is similar to a quadratic, but our b term is x^3 instead of x , so set $m = x^3$. That means that $m^2 = (x^3)^2 = x^6$. |
| $m^2 - 9m + 8 = 0$ | Find a pair of numbers that Add to -9 and Multiply to 8 : -1 and -8 . |
| $(m - 1)(m - 8) = 0$ | Factor. |
| $(x^3 - 1)(x^3 - 8) = 0$ | Replace $m = x^3$ |
| $(x^3 - 1) = 0$ or $(x^3 - 8) = 0$ | Set each factor equal to zero. |
| $x^3 = 1$ or $x^3 = 8$ | Bring the constant terms over to the right. |
| $x = 1$ or 2 | Take the cube root of both sides. |

1.10 Chapter 11 Exponents, Exponential Growth, and Imaginary Numbers Answers

Explanations for Chapter 11 Practice Drill 1 - Exponent Rules

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1. $(x^5)(x^8) = x^{5+8} = x^{13}$ When multiplying, add the exponents. $a = 13$.
-
2. $\frac{x^9}{x} = \frac{x^9}{x^1}$ Anything without an exponent has a power of 1.
 $\frac{x^9}{x^1} = x^{9-1} = x^8$ When dividing, subtract the exponents. $a = 8$.
-
3. $(x^5)^8 = x^{5(8)} = x^{40}$ When raising an exponent to another power, multiply the exponents. $a = 40$.
-
4. $\frac{(x^7)(x^3)}{x^4} = \frac{x^{10}}{x^4}$ Start with the top of the fraction. When multiplying, add the exponents, so $(x^7)(x^3) = x^{10}$.
 $x^{10-4} = x^6$ When dividing, subtract the exponents. $a = 6$.
-
5. $\left(\frac{(x^6)(x^{10})}{x^4}\right)^3 = \left(\frac{x^{16}}{x^4}\right)^3$ Start with the inside top of the fraction. When multiplying, add the exponents, so $(x^6)(x^{10}) = x^{6+10} = x^{16}$.
 $(x^{16-4})^3 = (x^{12})^3$ When dividing, subtract the exponents.
 $x^{12(3)} = x^{36}$ When raising an exponent to another power, multiply the exponents. $a = 36$.
-
6. $x^a(x^5) = (x^9)^{12}$ Start with the left side.
 $x^{a+5} = (x^9)^{12}$ When multiplying, add the exponents.
 $x^{a+5} = x^{(9)12}$ On the right side, when raising an exponent to another power, multiply the exponents.
 $x^{a+5} = x^{108}$ Simplify.
 $a + 5 = 108$ Set the exponents on each side of the equation equal to each other.
 $a = 103$ Subtract 5 from both sides.
-
7. $\frac{x^8}{x^{-4}} = x^{8-(-4)}$ When dividing, subtract the exponents.
 x^{12} $8 - (-4) = 8 + 4 = 12$, and $a = 12$.
-
8. $(x^{-4})(x^3)^5 = (x^{-4})(x^{3(5)})$ Start with the term on the right. When raising an exponent to another power, multiply the exponents.
 $(x^{-4})x^{15} = x^{-4+15} = x^{11}$ When multiplying, add the exponents. $a = -19$.
-
9. $\left(\frac{x^{-2}}{x^6}\right)(x^2)^8 = \left(\frac{x^{-2}}{x^6}\right)x^{2(8)}$ Start with the term on the right. When raising an exponent to another power, multiply the exponents.
 $(x^{-2-6})x^{16}$ When dividing, subtract the exponents.
 $(x^{-8})x^{16} = x^{-8+16} = x^8$ When multiplying, add the exponents. $a = 8$.

- $x^5 = 3$ and $a = x^{15}$
 $a = x^{5(3)} = (x^5)^3$
 $a = (3)^3 = 27$
10. $x = 3^{\frac{1}{5}}$
- $a = x^{15} = \left(3^{\frac{1}{5}}\right)^{15}$
 $a = \left(3^{\frac{1}{5}(15)}\right) = 3^{\frac{15}{5}} = 3^3 = 27$
- Let's see how we can use that x^5 . Rewrite $a = x^{15}$ using the fact that $15 = 5(3)$.
- When raising an exponent to a power, multiply the exponents. We can also go the other way: if the exponents are multiplied, that's the same as raising our exponent to another power.
- Plug in $x^5 = 3$ and simplify.
- OR
- If you don't see a way to use $x^5 = 3$ and get directly to x^{15} , solve for x . Take the 5th root of both sides. Or, put another way, raise both sides by the $\frac{1}{5}$ th power:
- $(x^5)^{\frac{1}{5}} = 3^{\frac{1}{5}}$, so $x^1 = 3^{\frac{1}{5}}$.
- Use $x = 3^{\frac{1}{5}}$ in the equation given.
- When raising an exponent to another power, multiply the exponents.

Explanations for Chapter 11 Practice Drill 2 - Radicals and Fractional Exponents

1. $x^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}}$
- $\frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$
- A negative exponent is a fraction: put the entire thing in the denominator.
- A $\frac{1}{2}$ exponent is a square root: the 1st power and the 2nd root. Answer choice (C).
-
2. $x^{\frac{2}{3}} = \sqrt[3]{x^2}$
- A fractional exponent is $\frac{\text{power}}{\text{root}}$, so the 2nd power and the 3rd root. Answer choice (B). We could also have written this as $(\sqrt[3]{x})^2$; it doesn't matter whether we do the power on the inside with the root outside, or vice versa. We just happened to have an answer with the power inside, so I matched with that one.
-
3. $x^{\frac{4}{3}} = x^{\frac{3}{3} + \frac{1}{3}}$
- $x^{\frac{3}{3}} x^{\frac{1}{3}}$
- $x^{\sqrt[3]{x}}$
- $x^{\frac{4}{3}} = \sqrt[3]{x^4}$
- $\sqrt[3]{x^3} \cdot x = x\sqrt[3]{x}$
- Split apart $\frac{4}{3}$.
- When multiplying, we add the exponents, which means that we can also the opposite: when exponents are added, we can rewrite the expression as multiplication.
- $x^{\frac{3}{3}} = x^1$. We can also rewrite the $x^{\frac{1}{3}}$ as $\sqrt[3]{x}$.
- OR
- Rewrite the $\frac{1}{3}$ fraction as a cubed root.
- Since each term is multiplied, split apart the cube root and simplify. Answer choice (H).

$$\sqrt{x^4(x^3)} = \sqrt{x^7}$$

$$(x^7)^{\frac{1}{2}}$$

$$x^{\frac{7}{2}}$$

$$x^{\frac{7}{2}} = x^{3+\frac{1}{2}}$$

$$x^{3+\frac{1}{2}} = x^3 x^{\frac{1}{2}}$$

$$x^3 \sqrt{x}$$

4. $x^{\frac{7}{2}} = \sqrt{x^7}$

$$\sqrt{x^7} = \sqrt{x^{2+2+2+1}}$$

$$\sqrt{x^{2+2+2+1}} = \sqrt{x^2 \cdot x^2 \cdot x^2 \cdot x}$$

$$\sqrt{x^2} \cdot \sqrt{x^2} \cdot \sqrt{x^2} \cdot \sqrt{x}$$

$$x \cdot x \cdot x \cdot \sqrt{x}$$

$$x^3 \sqrt{x}$$

Simplify the inside of the square root. When you multiply, add the exponents: $4 + 3 = 7$.

Rewrite the square root as an exponent.

When you raise an exponent to another power, multiply the exponents: $7 \cdot \frac{1}{2} = \frac{7}{2}$.

Simplify the exponent by breaking apart the fraction into a mixed number: $\frac{7}{2} = 3\frac{1}{2}$.

When multiplying, add the exponents. Here, we're using that rule in reverse: since the exponents 3 and $\frac{1}{2}$ are added, we can rewrite $x^{3+\frac{1}{2}}$ as multiplication.

Rewrite the remaining $x^{\frac{1}{2}}$ as \sqrt{x} , leaving $x^3 \sqrt{x}$, answer choice (F).

OR

If those last couple of steps were confusing, there's another way to work through it: rewrite the $\frac{1}{2}$ exponent as a square root.

Break apart x^7 using as many x^2 terms as we can: since $2 + 2 + 2 + 1 = 7$, we can break x^7 up into $x^{2+2+2+1}$. Those twos will be nice when we have to take the square root in a bit.

Rewrite the addition in the exponents as multiplication.

We can always break apart multiplication under a square root as separate square roots.

Square roots and squares cancel each other out: $\sqrt{x^2} = x$.

Combine the three x terms to x^3 , answer choice (F). If this seems like more steps than you're used to, that's fine: you can go directly from $\sqrt{x^7} = \sqrt{x^6} \sqrt{x} = x^3 \sqrt{x}$ once you have the pieces down. I've written out each step here because some of them can be a bit confusing, since it's the opposite way we've been using the exponent rules before now.

5. $x^{\frac{1}{5}} x^{\frac{1}{2}}$
 $x^{\frac{1}{5} + \frac{1}{2}}$
 $x^{\frac{7}{10}}$

Rewrite \sqrt{x} as an exponent: $\sqrt{x} = x^{\frac{1}{2}}$

When multiplying, add the exponents.

Give the fractions a common denominator to add them: $\frac{1}{5} + \frac{1}{2} = \frac{2}{10} + \frac{5}{10} = \frac{7}{10}$. Answer choice (E).

6. $x^{\frac{1}{3}} x^{\frac{1}{2}}$
 $x^{\frac{1}{3} + \frac{1}{2}}$
 $x^{\frac{5}{6}}$
 $x^{\frac{5}{6}} = \sqrt[6]{x^5}$

Rewrite the cube root as a $\frac{1}{3}$ power and the square root as a $\frac{1}{2}$ power.

When multiplying, add the exponents.

Give the fractions a common denominator to add them: $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$

Rewrite the fractional exponent using $\frac{\text{power}}{\text{root}}$. Answer choice (I).

7. $\frac{\sqrt[3]{x^5}}{x}$
 $x^{\frac{5}{3}-1}$
 $x^{\frac{2}{3}}$
 $x^{\frac{2}{3}} = \sqrt[3]{x^2}$

Rewrite the exponent on the top as a fraction.

When dividing, subtract the exponents.

Subtract $\frac{5}{3} - 1 = \frac{5}{3} - \frac{3}{3} = \frac{2}{3}$.

Rewrite the fractional exponent using $\frac{\text{power}}{\text{root}}$: the 2nd power and the 3rd root. Answer choice (B).

8. $(16x^8)^{\frac{1}{2}} = 16^{\frac{1}{2}} (x^8)^{\frac{1}{2}}$
 $4x^4$
- Distribute the $\frac{1}{2}$ power to each part.
 Start with $16^{\frac{1}{2}} = \sqrt{16} = 4$. For $(x^8)^{\frac{1}{2}}$, multiply the exponents: $x^{\frac{8}{1} \cdot \frac{1}{2}} = x^4$.
 Answer choice (K).
-
9. $\frac{x^{\frac{5}{2}}}{\sqrt[3]{x^9}} = \frac{x^{\frac{5}{2}}}{x^{\frac{9}{3}}}$
 $\frac{x^{\frac{5}{2}}}{x^3}$
 $x^{\frac{5}{2}-3}$
 $x^{-\frac{1}{2}}$
 $x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$
- Rewrite the root in the denominator as an exponent: $\sqrt[3]{x^9} = x^{\frac{9}{3}}$.
 Simplify the exponent in the denominator: $\frac{9}{3} = 3$.
 When dividing, subtract the exponents.
 Simplify: $\frac{5}{2} - 3 = \frac{5}{2} - \frac{6}{2} = -\frac{1}{2}$.
 A negative exponent is a fraction, and a $\frac{1}{2}$ exponent is a square root. Answer choice (C).
-
10. $\sqrt[3]{4x^3} (\sqrt[3]{2x}) (2\sqrt[3]{x^2}) = 2\sqrt[3]{4x^3 \cdot 2x \cdot x^2}$
 $2\sqrt[3]{8x^6}$
 $\sqrt[3]{8}\sqrt[3]{x^6}$
 $2(2x^2)$
 $4x^2$
- Since everything (except for the 2 near the end) is cube rooted, we can combine all of the cube roots. Bring the 2 out front and put everything else under a single cube root.
 Combine terms. $4(2) = 8$ and $x^3 \cdot x \cdot x^2 = x^{3+1+2} = x^6$.
 Break apart the cube root again now that everything underneath has been combined.
 The cube root of 8 is 2, since $2^3 = 8$, and $\sqrt[3]{x^6} = x^{\frac{6}{3}} = x^2$.
 Simplify. Answer choice (J).

Explanations for Chapter 11 Practice Drill 3 - Imaginary Numbers

1. $(3 + 12i) + (8 + 2i) = 11 + 14i$
- Combine like terms.
-
2. $(7 - 8i) + (12 + 7i) = 19 - i$
- Combine like terms: $7 + 12 = 19$ and $-8i + 7i = 1i = i$.
-
3. $(12 + 4i) - (5 - 3i) = 12 + 4i - 5 + 3i$
 $7 + 7i$
- Distribute the negative to each term in the second parenthesis. $-(-3i) = 3i$.
 Combine like terms.
-
4. $(4 + i) + (5 + i^2) = 9 + i + i^2$
 $9 + i - 1 = 8 + i$
- Combine like terms.
 Since $i = \sqrt{-1}$, $i^2 = (\sqrt{-1})^2 = -1$. Simplify.
-
5. $(-i)^2 - i^2 = i^2 - i^2$
 $i^2 - i^2 = 0$
- Start with the first $(-i)^2$. Since any negative number times itself is positive, $(-i)^2 = i^2$. Or, put another way, $(-i)^2 = ((-1)(i))^2 = (1)(i^2) = i^2$.
 Any number minus itself is zero. You could also expand out each i^2 , so $i^2 - i^2 = -1 - (-1) = -1 + 1 = 0$.
-
6. $8i(3i - 2) = 24i^2 - 16i$
 $-24 - 16i$
- Distribute the $8i$.
 Since $i = \sqrt{-1}$, $i^2 = (\sqrt{-1})^2 = -1$, and $24i^2 = -24$.

-
7. $(3 + 2i)(5 + 4i) = 15 + 12i + 10i + 8i^2$ Since we're multiplying two binomials, expand with FOIL: First, Outer, Inner, Last.
- $15 + 22i + 8i^2$ Combine like terms.
- $15 + 22i - 8$ Since $i = \sqrt{-1}$, $i^2 = (\sqrt{-1})^2 = -1$, and $8i^2 = -8$.
- $7 + 22i$ Simplify.
-
8. $(2 - 5i)(3 + 6i) = 6 + 12i - 15i - 30i^2$ Since we're multiplying two binomials, expand with FOIL: First, Outer, Inner, Last. Don't forget to distribute the negative when multiplying $-5i$!
- $6 - 3i - 30i^2$ Combine like terms.
- $6 - 3i - (-30)$ Since $i = \sqrt{-1}$, $i^2 = (\sqrt{-1})^2 = -1$.
- $36 - 3i$ Since $-(-30) = 30$, combine like terms.
-
9. $(1 - i)(1 + i) = 1 + i - i - i^2 = 1 - i^2$ Expand using FOIL. You may have also recognized this as the difference of squares, which gets you right to $(1 - i)(1 + i) = 1^2 - i^2 = 1 - i^2$.
- $1 - (-1) = 1 + 1 = 2$ Since $i = \sqrt{-1}$, $i^2 = (\sqrt{-1})^2 = -1$. Simplify.
-
10. $12i^3 + 6 = -12i + 6 = 6 - 12i$ Break apart $i^3 = i^2(i) = -1(i) = -1$.

Explanations for Chapter 11 Practice Drill 4 - Linear and Exponential Functions

-
1. A) Increasing linear Since the same number of books are added each month, this is a linear function. The slope is 20 books per month, going up by 20 every single month.
-
2. C) Increasing exponential The number of visitors each month doubles, which means it doesn't increase by the same amount each time, the amount it increases by keeps getting bigger and bigger. We are multiplying the number of visitors by 2 every single month, an exponential function.
-
3. A) Increasing linear Since the same number of people join each month, this is a linear function. The slope is 12 members per year, the same number of people joining each year, a nice straight line.
-
4. D) Decreasing exponential The population decreases by a percentage each year, rather than by the same amount each year. When there's a smaller population, fewer people will leave. It will always be 9% smaller each year, but 9% of a smaller number each time. The graph of population would curve down as it continues to the right, rather than being a straight line.
-
5. D) Decreasing exponential The computer's price goes down by a percentage, rather than by the same amount each year. Every year, as the price is lower, it decreases by the same percentage, which is a percentage of a smaller number, and therefore a smaller dollar decrease in price. The graph would be a curved line.
-

6. A) Increasing linear The price goes up by the exact same amount, \$10, each year. The graph of the price would be a straight line.
-
7. B) Decreasing linear The speed of the truck is decreasing by the same amount each second: 3 mph. The graph of the speed would be a straight line with a slope of -3 .
-
8. D) Decreasing exponential The size of the wetland isn't decreasing by the same amount; it is decreasing by the same fraction. Every decade there is 50% as much wetland as there was the decade before.
-
9. B) Decreasing linear The value of the company goes down by the exact same amount every single decade. When another 10 years pass? It'll go down by 1.2 million dollars again. The graph of the value of the company would be a straight line.
-
10. C) Increasing exponential The number of voles goes up by more and more each decade. If there were 100 voles in 2000, there were 300 voles in 2010. In 2020, there were 900. Every 10 years, the number triples. The graph would look like a curved line, curving up very quickly.

Explanations for Chapter 11 Practice Drill 5 - Exponential Functions Formula

1. $y = 30(2)^x$ Using the formula $y = ab^x$, the initial amount a is 30. The number of bacteria double each hour, so we're multiplying by 2, and $b = 2$.
-
2. $y = 350(0.95)^x$ Using the formula $y = ab^x$, the initial amount a is \$350. The cost decreases by 5%, which means the price each year is $100\% - 5\% = 95\%$ of the old price, so $b = 0.95$.
-
3. $y = 34.9(1.03)^x$ Using the formula $y = ab^x$, the initial amount a is 34.9 million. Since the question says that y is in millions, we can leave out that million part and $a = 34.9$. The company's value increases by 3% every year, so each year the value of the company will be 103% of what it was the year before, and $b = 1.03$.
-
4. $y = 12(3)^x$ Using the formula $y = ab^x$, the initial amount a is 12 fish. The number of fish tripled every year, so $b = 3$.
-
5. $y = 12(0.2)^x$ Using the formula $y = ab^x$, the initial amount a is 12 ounces of liquid meal (yum). The meal is digested at a rate of 80% per hour, so after a single hour only 20% of the meal will be left, and $b = 0.2$.
-
6. $y = 65000(1.06)^x$ Using the formula $y = ab^x$, the initial amount a is \$65,000 per year. The rate of increase is 6% per year, so each year the employee's salary will be 106% of what it was the previous year, and $b = 1.06$.
-
7. $y = 1664(0.5)^x$ Using the formula $y = ab^x$, the initial amount a is 1664 boxes. Since the number of boxes goes down by $\frac{1}{2}$ each day, $b = \frac{1}{2} = 0.5$.
-

-
8. $y = 23(1.023)^x$ Using the formula $y = ab^x$, the initial amount a is \$23 per share. That price has increased each year since 2010 by 2.3%, which means that in 2011 the price was 102.3% of what it was in 2010, and every year it's $100\% + 2.3\% = 102.3\%$ of the previous year's price, and $b = 1.023$.
-
9. $y = 4(0.5)^x$ Using the formula $y = ab^x$, the initial amount a is 4 milligrams of medication. The amount after a week is $\frac{1}{2}$ what it was, and $b = \frac{1}{2} = 0.5$.
-
10. $y = 760(0.99988)^x$ Using the formula $y = ab^x$, the initial amount a is 760 mmHg. The pressure decreases by 0.012% for each increase in 1 meter, so the pressure at 1 meter is $100\% - 0.012\% = 99.988\%$ of the pressure at a height of 0 meters. Convert 99.988% to a decimal, and $b = 0.99988$.

Explanations for Chapter 11 Practice Drill 6 - Time Intervals

-
1. 3 times per day Since the exponent is *multiplied* by 3, the value is increasing by 18% *multiple* times per day.
-
2. 2 times per year Since the exponent is *multiplied* by 2, the value is increasing by 3% *multiple* times per year.
-
3. Once per 5 years Since the exponent is *divided* by 5, the value decreases by 7% *once per* 5 years. When $t = 5$, the exponent is 1.
-
4. 12 times per year Since the exponent is *multiplied* by 12, the number of organisms triples *multiple* times per year.
-
5. Once per 898 years Since the exponent is *divided* by 898, the radioactive mass remaining decreases by a half *once per* 898 years. When $t = 898$, the exponent is 1.
-
6. $y = 18(2)^{\frac{x}{7}}$ The initial number of goldfish a is 18. Since the number of goldfish doubles every 7 years, $b = 2$. The number of goldfish doubles *once per* seven years, so the exponent needs to be divided by 7.
-
7. $y = 40(3)^{4x}$ The initial number of bacteria a is 40. The bacteria are tripling, so $b = 3$. They triple *multiple* times per hour, so we need to multiply the exponent by 4.
-
8. $y = 29(1.11)^{\frac{x}{2}}$ The initial value of the stock is \$29, so $a = 29$. The value increases by 11%, so $b = 1.11$. Since it happens *once per* 2 hours, the exponent needs to be divided by 2.
-
9. $y = 650,000(0.995)^{3x}$ The initial value of the house is \$650,000, so $a = 650,000$. The value of the house decreases by 0.5%, so the price becomes $100\% - 0.5\% = 99.5\%$ of what it was previously, and $b = 0.995$. Since this happens *multiple* times every year, we need to *multiply* the exponent by 3.
-
10. $y = 9\left(\frac{1}{2}\right)^{4x}$ The initial sample is 9 grams, so $a = 9$. It decays by $\frac{1}{2}$, so $b = \frac{1}{2}$. It decays *multiple* times per hour, so we need to multiply the exponent by 4.

Explanations for Chapter 11 Practice Drill 7 - Exponential Graphs

-
1. $y = 3^0 = 1$ Start by finding the y -intercept, where $x = 0$. Now we know the graph has to include the point $(0, 1)$. So it must be either (B) or (D). (It can't be (E) or (F) because it is an increasing exponential.)
- $y = 3^1 = 3$ Now find the point where $x = 1$. So, $(1, 3)$ has to be on the graph, which is answer choice (B).
-
2. $y = 2^0 = 1$ Start by finding the y -intercept, where $x = 0$. Now we know the graph has to include the point $(0, 1)$. So it must be either (B) or (D). (It can't be (E) or (F) because it is an increasing exponential.)
- $y = 2^1 = 2$ Now find the point where $x = 1$. So, $(1, 2)$ has to be on the graph, which is answer choice (D).
-
3. $y = 2(3)^0 = 2(1) = 2$ Start by finding the y -intercept, where $x = 0$. Now we know the graph has to include the point $(0, 2)$. So it must be answer choice (C).
-
4. $y = 3(2)^0 = 3(1) = 3$ Start by finding the y -intercept, where $x = 0$. Now we know the graph has to include the point $(0, 3)$, and it must be answer choice (A).
-
5. $y = 2^{-0} = 1$ Start by finding the y -intercept, where $x = 0$. Now we know the graph has to include the point $(0, 1)$. It's also decreasing because of the negative exponent, so the answer is either (E) or (F).
- $y = 2^{-(-1)} = 2^1 = 2$ The easiest point to compare between (E) and (F) is the different values at $x = -1$, so try out -1 , which gives us the point $(-1, 2)$ and answer choice (F).
-
6. $y = 3^{-0} = 1$ Start by finding the y -intercept, where $x = 0$. Now we know the graph has to include the point $(0, 1)$. It's also decreasing because of the negative exponent, so the answer is either (E) or (F).
- $y = 3^{-(-1)} = 3^1 = 3$ The easiest point to compare between (E) and (F) is the different values at $x = -1$, so try out -1 , which gives us the point $(-1, 3)$ and answer choice (E).

1.11 Chapter 12 Functions and Polynomials Answers

Explanations for Chapter 12 Practice Drill 1 - Inputs

-
1. $f(5) = 5(5+3)$ To find $f(5)$, replace every x in the definition for $f(x)$ with 5.
 $f(5) = 5(8)$
 $f(5) = 40$
-
2. $h(-2) = (-2)^2 + 6(-2)$ To find $h(-2)$, replace every x in the definition for $h(x)$ with -2 .
 $h(-2) = 4 - 12 = -8$
-
3. $f(10) = 8(10+1) - 10$ To find $f(10)$, replace every x in the definition for $f(x)$ with 10.
 $f(10) = 8(11) - 10$
 $f(10) = 88 - 10 = 78$
-
4. $g(-1) = (-1)^3 - (-1)^2 + (-1)$ To find $g(-1)$, replace every x in the definition for $g(x)$ with -1 .
 $g(-1) = -1 - 1 - 1 = -3$
-
5. $f(6) = 6^2 + 6$ First let's find $f(6)$ by replacing each x in the definition for $f(x)$ with 6.
 $f(6) = 36 + 6 = 42$
 $f(2) = 2^2 + 2$ Now let's find $f(2)$ by replacing each x with 2.
 $f(2) = 4 + 2 = 6$
 $f(6) - f(2) = 42 - 6 = 36$ Now we can find $f(6) - f(2)$.
-
6. $f(4) = 2^4 = 16$ First find $f(4)$ by replacing x in the definition for $f(x)$ with 4.
 $f(1) = 2^1 = 2$ Now find $f(1)$.
 $f(4) - f(1) = 16 - 2 = 14$ Now we can find $f(4) - f(1)$.
-
7. $h(11) = \frac{11-5}{2}$ First let's find $h(11)$ by replacing the x in the definition of $h(x)$ with 11.
 $h(11) = \frac{6}{2} = 3$
 $h(3) = \frac{3-5}{2}$ Now find $h(3)$.
 $h(3) = \frac{-2}{2} = -1$
 $h(11) + h(3) = 3 + -1 = 2$ Find $h(11) + h(3)$.
-
8. $g(1) = 1^2 + 3(1) - 4$ Since a is any integer between 0 and 4, it's either 1, 2, or 3. I'm going to use 1, because it's 1. It's such a tiny little guy. Easy to do math with.
 $g(1) = 1 + 3 - 4 = 0$ And 0 is a possible answer. If $a = 2$, then $g(2) = 6$, and if $a = 3$ then $g(3) = 14$. So possible values of $g(a)$ are 0, 6, or 14.
-

-
9. $g(2) = 4f(2)$ To find $g(2)$, start with what we know about the function g : $g(x) = 4f(x)$. Rewrite that equation, but replace each x with a 2. Looks like we'll need to find out the value of $f(2)$.
- $f(2) = 2^2 + 3$ To find $f(2)$, use the definition of $f(x)$, but replace the x with a 2.
- $f(2) = 4 + 3 = 7$ Now that we know $f(2) = 7$, we can replace that in the equation we wrote earlier.
- $g(2) = 4(7) = 28$ Replace $f(2)$ with 7.
-
10. $f(2x) = f(8)$ This is a weird definition of functions the SAT will sometimes use. You don't see it too much in math class: it's really just something the SAT likes to test to try to make functions more confusing. We want to find $f(8)$, but the definition for f is a little odd: it's the definition of $f(2x)$. That means that $x = 4$ to find $f(8)$, so we have $f(2(4)) = f(8)$.
- $f(2(4)) = 4 + 1 = 5$ Plug in $x = 4$ to the definition of f , and you'll get $f(8) = 5$.

Explanations for Chapter 12 Practice Drill 2 - Outputs

-
1. $-23 = 5x - 8$ Since $h(x) = -23$, rewrite the function but replace $h(x)$ with -23 .
- $-15 = 5x$ Add 8 to both sides.
- $-3 = x$ Divide both sides by 5. $h(-3) = -23$.
-
2. $76 = 3x^2 + 1$ Since $f(x) = 76$, rewrite the function but replace $f(x)$ with 76.
- $75 = 3x^2$ Subtract 1 from both sides.
- $25 = x^2$ Divide both sides by 3.
- $\pm 5 = x$ Take the square root of both sides. Any time a variable is squared, there are two possible solutions. In this case, we only care about the positive one, 5. $f(5) = 76$.
-
3. $7 = 3(x+5)^2 + 7$ Since $f(x) = 7$, rewrite the function but replace $f(x)$ with 7.
- $0 = 3(x+5)^2$ Subtract 7 from both sides.
- $0 = (x+5)^2$ Divide both sides by 3.
- $0 = x + 5$ Take the square root of both sides.
- $-5 = x$ Subtract 5 from both sides. $f(-5) = 7$. You may have also noticed that this is a parabola in vertex form, with a vertex at $(-5, 7)$, which also gives us $x = -5$.
-
4. $f(b) = -2(b-4)$ To find $f(b)$, replace x in the definition of $f(x)$ with a b .
- $10 = -2(b-4)$ Since $f(b) = 10$, replace $f(b)$ with 10.
- $10 = -2b + 8$ Distribute the -2 .
- $2 = -2b$ Subtract 8 from both sides.
- $-1 = b$ Divide both sides by -2 . $f(-1) = 10$.
-

-
- $g(m) = m(m - 5)$
 $24 = m(m - 5)$
 $24 = m^2 - 5m$
 5. $0 = m^2 - 5m - 24$
 $(m - 8)(m + 3)$
 $m = 8$ or $m = -3$
- To find $g(m)$, replace each x in the function $g(x)$ with an m .
 Since $g(m) = 24$, replace $g(m)$ with 24.
 Distribute the m on the right side of the equation.
 To solve the quadratic, subtract 24 from both sides.
 We need two numbers that add to -5 and multiply to -24 , so the factors are $m - 8$ and $m + 3$.
 Solve for each solution by setting each factor equal to zero: $m - 8 = 0$, or $m + 3 = 0$.

Explanations for Chapter 12 Practice Drill 3 - Graphing Functions

-
1. $f(2) = 0$
 To find $f(2)$, we want to know the y -coordinate where $x = 2$ on the function $f(x)$. (Since 2 is inside the parentheses, it's replacing x .) That's the point $(2, 0)$, and $f(2) = 0$.
-
2. $f(0) = 3$
 To find $f(0)$, we want to know the y -coordinate where $x = 0$ on the function $f(x)$. (Since 0 is inside the parentheses, it's replacing x .) That's the point $(0, 3)$, and $f(0) = 3$.
-
3. $f(-3) = 0$
 To find $f(-3)$, we want to know the y -coordinate where $x = -3$ on the function $f(x)$. (Since -3 is inside the parentheses, it's replacing x .) That's the point $(-3, 0)$, and $f(-3) = 0$.
-
4. $g(0) = 3$
 To find $g(0)$, we want to know the y -coordinate where $x = 0$ on the function $g(x)$. (Since 0 is inside the parentheses, it's replacing x .) That's the point $(0, 3)$, and $g(0) = 3$.
-
5. $g(-5) = 3$
 To find $g(-5)$, we want to know the y -coordinate where $x = -5$ on the function $g(x)$. (Since -5 is inside the parentheses, it's replacing x .) That's the point $(-5, 3)$, and $g(-5) = 3$.
-
6. $g(-1) = 4$
 The maximum value is just the highest y -coordinate for function g , which is at the point $(-1, 4)$. The maximum value is 4.
-
7. $g(4) = -3$
 If $g(a) = -3$, then there is some point with an x -coordinate of a (inside the parentheses) and a y -coordinate of -3 (outside the parentheses). Looking at the graph of g , there's only one place where the graph has $y = -3$: when $x = 4$, and $a = 4$.
-
8. $p(3) = g(3) + 1$
 $g(3) = 0$
 $p(3) = 0 + 1$
 To find $p(3)$, start by writing out the definition for $p(x)$ and replacing each x with 3. To find $p(3)$, it looks like we'll need to find $g(3)$.
 Find the point on the graph of g where $x = 3$. The point $(3, 0)$ means that $g(3) = 0$.
 Replace $g(3)$ with 0 in our equation from above, and $p(3) = 1$.
-

- 9.
- | | |
|-------------------------|--|
| $h(-2) = g(-2) + f(-2)$ | To find $h(-2)$, start by writing out the definition for $h(x)$ given, but replace each x with a -2 . It looks like to $h(-2)$ we'll need to know $g(-2)$ and $f(-2)$. |
| $f(-2) = 4$ | To find $f(-2)$, we want to know the y -coordinate where $x = -2$ on the function $f(x)$. (Since -2 is inside the parentheses, it's replacing x .) That's the point $(-2, 4)$, and $f(-2) = 4$. |
| $g(-2) = 3$ | Now, to find $g(-2)$, we want to know the y -coordinate where $x = -2$ on the function $g(x)$. (Since -2 is inside the parentheses, it's replacing x .) That's the point $(-2, 3)$, and $g(-2) = 3$. |
| $h(-2) = 3 + 4 = 7$ | Now replace $g(-2)$ with 3 and $f(-2)$ with 4 in the equation from our first step, and $h(-2) = 7$. |

- 10.
- | | |
|-------------------|--|
| $f(0) = g(0) = 3$ | We want to find some value of x so that $f(x)$ is the same as $g(x)$. In other words, for some x -coordinate, both graphs have the exact same y -coordinate. Try different points until you find one that works: For instance, if we try $x = -3$, then $f(-3) = 0$, but $g(-3) = 2$. The only place where both functions match up is at $x = 0$, where both $f(0)$ and $g(0)$ are 3. |
|-------------------|--|

Explanations for Chapter 12 Practice Drill 4 - Tables and Functions

- 1.
- | | |
|-------------|---|
| $f(1) = -1$ | Since 1 is inside the parentheses, that's our x . Check the table in the x -column for 1. (The 4th row below the header.) Now go over to the $f(x)$ column, which gives us a -1 , and $f(1) = -1$. |
|-------------|---|
- 2.
- | | |
|--------------|--|
| $g(-1) = -2$ | Since -1 is inside the parentheses, that's our x . Check the table in the x -column for -1 . (The 2nd row below the header.) Now head over to the column for $g(x)$, and $g(-1) = -2$. |
|--------------|--|
- 3.
- | | |
|-------------|--|
| $f(0) = 0$ | To find $f(0) + g(-2)$, start by finding $f(0)$. Since 0 is inside the parentheses, that's our x . Check the table in the x -column for 0 (3 rows down from the x), then check the $f(x)$ column, and since there's a 0 there, $f(0) = 0$. |
| $g(-2) = 2$ | There's a -2 inside the parentheses, so $x = -2$. That's the row right near the top, and looking over at the $g(x)$ column, there's a 2, so $g(-2) = 2$. |
| $0 + 2 = 2$ | Now we can find $f(0) + g(-2)$. |
- 4.
- | | |
|-------------|---|
| $g(0) = -1$ | We want to know when $g(x)$ (the column all the way to the right) is equal to -1 . That is right in the middle, when $x = 0$, so $g(0) = -1$. |
|-------------|---|
- 5.
- | | |
|-------------------|--|
| $b = f(-1) = 1$ | Start by finding $f(-1)$. The -1 is inside the parentheses, so that's our x -value. In the table, go to where $x = -1$, then look at the $f(x)$ column, and $f(-1) = 1$, so $b = 1$. |
| $g(b) = g(1) = 2$ | Now use the value of $b = 1$ to find $g(b) = g(1)$. Since there's a 1 inside the parentheses, $x = 1$. Looking at the table, when $x = 1$, then $g(1) = 2$. |

Explanations for Chapter 12 Practice Drill 5 - Roots

-
1. $0 = x(x+4)(x-2)$ The equation is in factored form, so we can use each factor to find the x -intercepts, the places it crosses the x -axis and $y = 0$.
- $x = 0, x + 4 = 0,$ and $x - 2 = 0$ Set each factor equal to zero.
- $x = 0, -4,$ and 2 Solve each equation.
-
2. $x + 3 = 0$ or $2x + 1 = 0$ Set each factor equal to zero.
- $x = -3$ For the first factor, subtract 3 from both sides, and one possible solution is $x = -3$.
- $2x = -1$ For the other factor, subtract 1 from both sides.
- $x = -\frac{1}{2}$ Divide both sides by 2, and the other solution is $x = -\frac{1}{2}$.
-
3. $(x^2 - 1) = 0$ or $(x + 5)^2 = 0$ Set each factor equal to zero to find the roots.
- $x^2 = 1$ Solve each equation. For the first one, add 1 to each side.
- $x = \pm 1$ Take the square root of both sides. Whenever a variable is squared, there are two solutions, positive and negative. You could solve $x^2 - 1 = 0$ by factoring it as the difference of squares, $(x + 1)(x - 1) = 0$, and $x = -1$ or 1 .
- $x + 5 = 0$ For the other equation, take the square root of both sides.
- $x = -5$ Subtract 5 from both sides. The roots are $-5, -1,$ and 1 .
-
4. $x^2(x^2 - 3x - 24)$ Since each term has an x^2 (or more) in it, start by factoring out x^2 .
- $x^2(x - 6)(x + 4)$ Now factor the quadratic inside the parentheses. Remember A.M. We need something that Adds to -2 and Multiplies to -24 , so the factors are $(x - 6)$ and $(x + 4)$.
- $x^2, (x - 6), (x + 4)$ We have now factored the polynomial completely.
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5. $ax^3 + bx^2 + cx + d = (x - 2)(x + 3)(x - 5)$ Since we know the roots of the polynomial, we know the factors. A root of 2 means that the polynomial has a factor $(x - 2)$, so $x - 2 = 0$ and $x = 2$. Similarly, a root of 3 means that the polynomial has a factor of $(x + 3)$, and a root of 5 means the polynomial has a factor of $(x - 5)$.
- $ax^3 + bx^2 + cx + d = \text{something something} + 30$ We could completely expand out $(x - 2)(x + 3)(x - 5)$ to find what d is (first we'd FOIL two of the factors together, and then multiply that by the remaining factor), but since we only need to know d , the last term, we can find it just by multiplying $(-2)(3)(-5)$ together to get $d = 30$. (Similarly, we know that $a = 1$ because the first terms will multiply together to give us x^3 .) If you're not sure why that works (and why it *doesn't* work for the b or c terms), try expanding out the factors on the right of the equation in the previous step. You should get $x^3 - 4x^2 - 11x + 30$.

Explanations for Chapter 12 Practice Drill 6 - Graphs of Roots

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1. $0 = x(x+2)(x-2)$ Let's find the roots or x -intercepts by setting $y = 0$. This graph has three x -intercepts. Set each factor to zero and solve, and $x = 0, x = -2,$ or $x = 2$. Since none of the factors are squared, the roots are all places the graph crosses the x -axis, so it must be graph (F).

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| 2. | $0 = x(x-1)(x+1)$ | Let's find the roots or x -intercepts by setting $y = 0$. This graph has three x -intercepts. Set each factor to zero and solve, and $x = 0$, $x = 1$, or $x = -1$. Since none of the factors are squared, the roots are all places the graph crosses the x -axis, so it must be graph (A). |
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| 3. | $0 = (x-2)^2(x+2)$ | Find the x -intercepts by setting $y = 0$. This graph has two x -intercepts, where $(x-2)^2 = 0$ and where $(x+2) = 0$. Solving each equation gives us the x -intercepts 2 and -2 . Since the factor $(x-2)$ is squared, the x -intercept we got from it, $x = 2$, is a place where the graph bounces against the x -axis. So we need a graph that bounces at $x = 2$ and crosses at $x = -2$, and the answer is (D). |
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| 4. | $0 = (x-2)(x+2)^2$ | Well, this is just the opposite of question 3. Same x -intercepts, -2 and 2, but this time the graph bounces from the $(x+2)^2$ term, at $x = -2$, and then crosses at $x = 2$, and the answer is (E). |
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| 5. | $0 = (x-1)^2(x+1)$ | Find the x -intercepts by setting $y = 0$. There are two x -intercepts, 1 and -1 . Since $(x-1)$ is squared, the x -intercept at $x = 1$ is a bounce in the graph, and the equation crosses the x -axis at $x = -1$, and the answer is (B). |
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| 6. | $0 = (x-1)(x+1)^2$ | This is the same as question 5, so take everything I said there, basically. The only difference is that this time the $(x+1)$ factor is squared, so the x -intercept at -1 is a bounce, and the x -intercept at 1 is a cross, so the answer is (C). |
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| 7. | $0 = x^2(x-1)(x+1)$ | Find the x -intercepts by setting $y = 0$ and solving to find where each factor equals 0. This time there are three factors: x , $(x-1)$, and $(x+1)$. That gives us 3 x -intercepts, 0, 1, and -1 . Since the x is squared, the graph bounces at $x = 0$, and the answer is (H). |
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| 8. | $0 = (x-2)^2(x+2)^2$ | There are two x -intercepts, 2 and -2 , and since they're both squared, they're both places where the graph bounces against the x -axis. The answer is (I). |
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| 9. | $0 = x^2(x+2)(x-2)$ | There are 3 x -intercepts: 0, -2 , and 2. Only the x factor is squared, so the graph will cross at -2 , bounce at 0, and then cross at 2, and the answer is (G). |
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Explanations for Chapter 12 Practice Drill 7 - Transformations

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| 1. | Flipped vertically and moved up 2 | If $h(x) = -f(x) + 2$, then there are 2 transformations: the negative sign in front of the $f(x)$ flips the function vertically over the x -axis, and the $+2$ moves the entire function up 2. |
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| 2. | Moved right 4 and down 2 | The -4 inside the parentheses moves the function to the <i>right</i> 4 units. The -2 outside the parentheses moves the function down 2 units. |
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| 3. | $-(3x^2 + 2x) = -3x^2 - 2x$
Flipped vertically | Since the second equation is just the first one multiplied by -1 , this is equivalent to the transformation $y = -f(x)$.
The transformation $y = -f(x)$ is a vertical flip of the function $f(x)$ over the x -axis. |
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4. 7 units right
 $f(x+3)^2+2$ is a parabola shifted left 3 units and up 2 units. $g(x) = (x-4)^2+2$ is a parabola that is also shifted up 2 units, but shifted to the right 4 units. Therefore, starting at f , we'd need to go right 3 units to get back to the y -axis, and then another 4 units right to get to g , for a total of 7 units to the right.
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5. $f(x+2)$
 The first equation, $y = (x+2)^3 + 5(x+2)^2$, is pretty similar to the second equation, $x^3 + 5x^2$. The first one just has an $(x+2)$ anywhere there's an x in the second equation.
 2 units left
 That means the first equation is equivalent to a transformation of $f(x+2)$, a translation 2 units to the left.
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6. $y = -5x^3$
 If $y = f(x)$ is flipped over the x -axis, it becomes $y = -f(x)$. So, to flip this equation vertically, just multiply it by -1 .
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7. $y = 5(x-7)^3 - 9$
 A translation 7 units right and 9 units down is the translation from $f(x)$ to $f(x-7) - 9$. The shift down is the -9 outside the function, and the -7 is the translation 7 units right, inside the function. So, any translation horizontally should replace the x , and instead of the x in $5x^3$, we now have $5(x-7)^3$. Moving the function down 9 units just takes subtracting 9 at the end, so $5(x-7)^3$ becomes $5(x-7)^3 - 9$.
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8. $f(x-3)+2$
 To move the function up 2 units, just add 2 outside the function. To move it left 3 units, subtract 3 inside the parentheses. Remember, anything inside the parentheses moves the function horizontally, anything outside the parentheses moves the function vertically.
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9. $g(x+0.5)+8$
 To move the function up 8 units, add 8 outside the function. To move it left 0.5 units, add 0.5 inside the parentheses.
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10. $f(x-2+5)+3-4$
 To move the function left 5 units, add 5 inside the parentheses. To move the function down 4 units, subtract 4 outside the parentheses.
 $f(x+3)-1$
 Simplify.

1.12 Chapter 13 Circles Answers

Explanations for Chapter 13 Practice Drill 1 - Area and Circumference

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1. $A = \pi 9^2$
 $A = 81\pi$
 Use the radius, 9, with $A = \pi r^2$.
 Simplify.
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2. $C = 2\pi(2)$
 $C = 4\pi$
 Use the radius, 2, with $C = 2\pi r$.
 Combine like terms.
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3. $A = \pi 10^2$
 $A = 100\pi$
 Since the diameter is 20 miles, the radius is 10 miles. Use that with $A = \pi r^2$.
 Simplify.

4.	$C = 80\pi$	Since the circle is 80 miles wide, the diameter is 80. Use that with $C = \pi d$.
5.	$A = \pi(\sqrt{2})^2$ $A = 2\pi$	Use the radius of $\sqrt{2}$ with the area formula $A = \pi r^2$. Simplify. $(\sqrt{2})^2 = 2$.
6.	$12\pi = 2\pi r$ $6 = r$	Use the circumference formula, $C = 2\pi r$, and plug in the circumference given. Divide both sides by 2π .
7.	$16\pi = \pi r^2$ $16 = r^2$ $4 = r$	Use the area formula, $A = \pi r^2$, and plug in the area given for A . Divide both sides by π . Take the square root of both sides. $\sqrt{16} = 4$.
8.	$16\pi = 2\pi r$ $8 = r$	Use the circumference formula, $C = 2\pi r$, and plug in the circumference given. Divide both sides by 2π .
9.	$25\pi = \pi r^2$ $25 = r^2$ $5 = r$ $10 = d$	Use the area formula, $A = \pi r^2$, and plug in the area given for A . Divide both sides by π . Take the square root of both sides. $\sqrt{25} = 5$. Since the question asks for the diameter, multiply the radius by 2.
10.	$2\pi = \pi r^2$ $2 = r^2$ $\sqrt{2} = r$	Use the area formula, $A = \pi r^2$, and plug in the area given for A . Divide both sides by π . Take the square root of both sides.

Explanations for Chapter 13 Practice Drill 2 - Arcs

1.	$\frac{12\pi}{32\pi} = \frac{x}{360}$ $\frac{3}{8} = \frac{x}{360}$ $8x = 3(360)$ $x = 135$	Set up the equation $\frac{\text{arc}}{\text{circumference}} = \frac{\text{degrees}}{360}$. Reduce the fraction. Cross multiply. Solve.
2.	$\frac{5\pi}{C} = \frac{30}{360}$ $\frac{5\pi}{C} = \frac{1}{12}$ $C = 5(12\pi)$ $C = 60\pi$	Set up the equation $\frac{\text{arc}}{\text{circumference}} = \frac{\text{degrees}}{360}$. Reduce the fraction. Cross multiply. Simplify.

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3. $\frac{a}{24\pi} = \frac{210}{360}$
 $\frac{a}{24\pi} = \frac{7}{12}$
 $a = 14\pi$
- Set up the equation $\frac{\text{arc}}{\text{circumference}} = \frac{\text{degrees}}{360}$
- Reduce the fraction.
- Either cross multiply and solve or find the multiplier: $12 \times \underline{2\pi} = 24\pi$ so $7 \times \underline{2\pi} = 14\pi$.
-
4. $\frac{a}{48\pi} = \frac{135}{360}$
 $\frac{a}{48\pi} = \frac{3}{8}$
 $a = 18\pi$
- Set up the equation $\frac{\text{arc}}{\text{circumference}} = \frac{\text{degrees}}{360}$.
- Reduce the fraction.
- Either cross multiply and solve or find the multiplier: $8 \times \underline{6\pi} = 48\pi$ so $3 \times \underline{6\pi} = 18\pi$.
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5. $C = 2\pi(3)$
 $C = 6\pi$
 $\frac{a}{6\pi} = \frac{60}{360}$
 $\frac{a}{6\pi} = \frac{1}{6}$
 $a = \pi$
- Start by finding the circumference, using the radius given and $C = 2\pi r$
- Set up the equation $\frac{\text{arc}}{\text{circumference}} = \frac{\text{degrees}}{360}$.
- Reduce the fraction.
- Either cross multiply and solve or find the multiplier: $6 \times \underline{\pi} = 6\pi$ so $1 \times \underline{\pi} = \pi$.
-
6. $\frac{1}{5} = \frac{x}{360}$
 $5x = 360$
 $x = 72$
- Set up the equation $\frac{\text{arc}}{\text{circumference}} = \frac{\text{degrees}}{360}$ Since you're given the ratio of the arc out of the circumference the left side of the equation is just that ratio: in this case, $\frac{1}{5}$.
- Cross multiply.
- Divide both sides by 5.
-
7. $C = 2\pi(6)$
 $C = 12\pi$
 $\frac{4\pi}{12\pi} = \frac{x}{360}$
 $\frac{1}{3} = \frac{x}{360}$
 $3x = 360$
 $x = 120$
- Start by finding the circumference, using the radius given and $C = 2\pi r$.
- Set up the equation $\frac{\text{arc}}{\text{circumference}} = \frac{\text{degrees}}{360}$.
- Reduce the fraction.
- Cross multiply (or find the multiplier between the two ratios).
- Divide both sides by 3.
-
8. $360 - 45 = 315$
 $\frac{a}{56\pi} = \frac{315}{360}$
 $\frac{a}{56\pi} = \frac{7}{8}$
 $8a = 392\pi$
 $a = 49\pi$
- Since we want arc CDA , find the angle for COA (the big one) by subtracting the small angle COA from 360.
- Set up the equation $\frac{\text{arc}}{\text{circumference}} = \frac{\text{degrees}}{360}$.
- Reduce the fraction.
- Cross multiply (or find the multiplier between the two ratios.)
- Divide both sides by 8.
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| 9. | $\frac{12}{C} = \frac{30}{360}$ $\frac{12}{C} = \frac{1}{12}$ $C = 144$ | Set up the equation $\frac{\text{arc}}{\text{circumference}} = \frac{\text{degrees}}{360}$
Reduce the fraction.
Cross multiply. |
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| 10. | $8\pi + 12\pi = 20\pi$ $\frac{8\pi}{20\pi} = \frac{x}{360}$ $\frac{2}{5} = \frac{x}{360}$ $5x = 2(360)$ $x = 144$ | Find the total circumference by adding up the 2 arcs.
Set up the equation $\frac{\text{arc}}{\text{circumference}} = \frac{\text{degrees}}{360}$.
Reduce the fraction.
Cross multiply (or find the multiplier between the two ratios).
Solve. |
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Explanations for Chapter 13 Practice Drill 3 - Radians

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| 1. | $\frac{x}{\pi} = \frac{45}{180}$ $\frac{x}{\pi} = \frac{1}{4}$ $4x = \pi$ $x = \frac{\pi}{4}$ | Use $\frac{\text{radians}}{\pi} = \frac{\text{degrees}}{180}$
Reduce the fraction.
Cross multiply.
Divide both sides by 4. |
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| 2. | $\frac{\frac{\pi}{6}}{\pi} = \frac{x}{180}$ $\frac{1}{6} = \frac{x}{180}$ $6x = 180$ $x = 30$ | Use $\frac{\text{radians}}{\pi} = \frac{\text{degrees}}{180}$
Reduce the fraction.
Cross multiply (or find the multiplier between the two ratios, 30).
Divide both sides by 6 |
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| 3. | $\frac{3\pi}{\pi} = \frac{x}{180}$ $\frac{3}{1} = \frac{x}{180}$ $x = 3(180) = 540$ | $\frac{\text{radians}}{\pi} = \frac{\text{degrees}}{180}$
Reduce the fraction.
Cross multiply. |
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| 4. | $\frac{\frac{11\pi}{6}}{\pi} = \frac{x}{180}$ $\frac{11}{6} = \frac{x}{180}$ $6x = 11(180)$ $x = 330$ | Use $\frac{\text{radians}}{\pi} = \frac{\text{degrees}}{180}$.
Reduce the fraction.
Cross multiply.
Divide both sides by 6 |
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| 5. | $\frac{x}{\pi} = \frac{720}{180}$ $\frac{x}{\pi} = 4$ $x = 4\pi$ | Use $\frac{\text{radians}}{\pi} = \frac{\text{degrees}}{180}$.
Reduce the fraction.
Multiply both sides by π . |
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6. $\frac{x}{\pi} = \frac{315}{180}$ Use $\frac{\text{radians}}{\pi} = \frac{\text{degrees}}{180}$.
 $\frac{x}{\pi} = 1.75$ Reduce the fraction.
 $x = 1.75\pi$ or $\frac{7\pi}{4}$ Multiply both sides by π .
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7. $\frac{9\pi}{\pi} = \frac{x}{180}$ Use $\frac{\text{radians}}{\pi} = \frac{\text{degrees}}{180}$.
 $9 = \frac{x}{180}$ Reduce the fraction.
 $x = 9(180) = 1,620$ Multiply both sides by 180.
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8. $\frac{2.9}{\pi} = \frac{x}{180}$ Use $\frac{\text{radians}}{\pi} = \frac{\text{degrees}}{180}$.
 $\pi x = 2.9(180)$ Cross multiply.
 $\pi x = 522$ Simplify.
 $x = 166.1579 = 166$ Divide both sides by π and round to the nearest whole number.
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9. $35 + 10 = 45$ Find the measure of angle ABD by adding up the two angles that make up ABD : ABC and CBD . (Try drawing the two angles, using the same points B and C in both angles, if you're not sure why.)
 $\frac{x}{\pi} = \frac{45}{180}$ Use $\frac{\text{radians}}{\pi} = \frac{\text{degrees}}{180}$.
 $\frac{x}{\pi} = \frac{1}{4}$ Reduce the fraction.
 $4x = \pi$ Cross multiply.
 $x = \frac{\pi}{4}$ Divide both sides by 4.
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10. $\frac{x}{\pi} = \frac{10}{180}$ Use $\frac{\text{radians}}{\pi} = \frac{\text{degrees}}{180}$.
 $\frac{x}{\pi} = \frac{1}{18}$ Reduce the fraction.
 $18x = \pi$ Cross multiply.
 $x = \frac{\pi}{18}$ Divide both sides by 18.

Explanations for Chapter 13 Practice Drill 4 - Equation of a Circle

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1. $(x-0)^2 + (y-0)^2 = 7^2$ Use $(x-h)^2 + (y-k)^2 = r^2$ with $h=0$, $k=0$, and $r=7$.
 $x^2 + y^2 = 49$ Simplify.
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2. $x^2 + y^2 = 4$ Use $(x-h)^2 + (y-k)^2 = r^2$ with $h=0$, $k=0$, and $r^2=4$ so $r=2$.
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3. $(x-2)^2 + (y-3)^2 = 64$ Use $(x-h)^2 + (y-k)^2 = r^2$.
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4. $(x-4)^2 + y^2 = 81$ Use $(x-h)^2 + (y-k)^2 = r^2$ with $h=4$, $k=0$, and $r^2=81$ so $r=9$.
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1.12 Chapter 13 Circles Answers and Explanations

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5. $(x-3)^2 + (y-(-6))^2 = 2^2$ Use $(x-h)^2 + (y-k)^2 = r^2$ with $h = 3$, $k = -6$, and $r = 2$.
 $(x-3)^2 + (y+6)^2 = 4$ Simplify.
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6. $(x-8)^2 + (y-2)^2 = 121$ Use $(x-h)^2 + (y-k)^2 = r^2$ with $h = 8$, $k = 2$, and $r^2 = 121$, so $r = 11$.
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7. $(x-1)^2 + (y+9)^2 = 10$ Use $(x-h)^2 + (y-k)^2 = r^2$ with $h = 1$, $k = -9$, and $r^2 = 10$ so $r = \sqrt{10}$.
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8. $(x-3)^2 + (y-0)^2 = \left(\frac{3}{2}\right)^2$ Use $(x-h)^2 + (y-k)^2 = r^2$ with $h = 3$, $k = 0$, and $r = \frac{3}{2}$.
 $(x-3)^2 + y^2 = \frac{9}{4}$ Simplify.
-
- $\left(\frac{-3+3}{2}, \frac{-4+4}{2}\right) = (0,0)$ Find the center of the circle by finding the midpoint of the diameter: $(0,0)$.
 $(3-0)^2 + (-4-0)^2 = r^2$ Solve for r^2 using Pythagorean theorem, where the base of the triangle is the difference between the x values and the height of the triangle is the difference between the two y values.
9. $9 + 16 = r^2$
 $25 = r^2$ Don't bother solving for r , since we only need r^2 .
 $(x-0)^2 + (y-0)^2 = 25$ Use $(x-h)^2 + (y-k)^2 = r^2$ along with the center $(0,0)$ and the the value of r^2 we found.
 $x^2 + y^2 = 25$ Simplify.
-
- $(13-1)^2 + (10-5)^2 = r^2$ Solve for r^2 using the Pythagorean Theorem, where the base of the triangle is the difference between the x values and the height of the triangle is the difference between the two y values.
10. $12^2 + 5^2 = r^2$ Simplify.
 $144 + 25 = 169 = r^2$ Don't bother solving for r , since we only need r^2 .
 $(x-1)^2 + (y-5)^2 = 169$ Use $(x-h)^2 + (y-k)^2 = r^2$.

Explanations for Chapter 13 Practice Drill 5 Complete the Square

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1. $(x-2)^2 + y^2 + 24y = 0 + 4$ Complete the square with the $x^2 - 4x$ terms: half of -4 is -2 , so rewrite the x terms as $(x-2)^2$ and add $(-2)^2 = 4$ to the right side to balance it out.
 $(x-2)^2 + (y+12)^2 = 4 + 144$ Now complete the $y^2 + 24y$ terms: half of 24 is 12 , so rewrite the y terms as $(y+12)^2$ and add $12^2 = 144$ to the right side to balance it out.
 $(x-2)^2 + (y+12)^2 = 148$ Use $(x-h)^2 + (y-k)^2 = r^2$ to find the center is at $(2, -12)$.
-
2. $x^2 + (y+1)^2 = 48 + 1$ The x^2 can't be simplified any farther, so move on to the $y^2 + 2y$ term. Half of 2 is 1 , so rewrite the y terms as $(y+1)^2$ and add 1^2 to the right side to balance it out.
 $x^2 + (y+1)^2 = 49$ Use $(x-h)^2 + (y-k)^2 = r^2$ to find that the center is at $(0, -1)$.
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3. $(x - 1.5)^2 + y^2 + y = 0 + 2.25$ Start with the $x^2 - 3x$ term. Half of 3 is 1.5, so rewrite the x terms as $(x - 1.5)^2$ and add $1.5^2 = 2.25$ to the right side to balance it out. (As always, squaring fractions is easier than squaring decimals, so you could rewrite 1.5 as $\frac{3}{2}$ and square that if this were on the no calculator section.)
- $(x - 1.5)^2 + (y + 0.5)^2 = 2.25 + 0.25$ Now complete the square for the $y^2 + y$. Half of 1 (since we have 1y) is 0.5, so rewrite the y terms as $(y + 0.5)$ and add $0.5^2 = 0.25$ to the right side to balance it out.
- $(x - 1.5)^2 + (y + 0.5)^2 = 2.5$ Use $(x - h)^2 + (y - k)^2 = r^2$ to find that the center is at $(1.5, -0.5)$.
-
4. $(x - 6)^2 + y^2 + 4y = 9 + 36$ Start by completing those $x^2 - 12x$ terms. Half of -12 is -6 , so rewrite those x terms as $(x - 6)^2$ and add $(-6)^2 = 36$ to the right side to balance it out. Oh, and move that -9 over to the right side too.
- $(x - 6)^2 + (y + 2)^2 = 45 + 4$ Now complete the $y^2 + 4y$ terms. Half of 4 is 2, so rewrite the y terms as $(y + 2)$ and add $2^2 = 4$ to the right side to balance it out.
- $(x - 6)^2 + (y + 2)^2 = 49$ Use $(x - h)^2 + (y - k)^2 = r^2$ to find that the $r^2 = 49$ and $r = 7$
-
5. $x^2 + 6x + y^2 + y = 3 + 9$ As always, let's start with those x terms: $x^2 + 6x$. Half of 6 is 3, so we can rewrite them as $(x + 3)^2$ and balance it out on the right side by adding $3^2 = 9$ to the right side.
- $x^2 + 6x + y^2 + y = 12 + 0.25$ Now deal with those $y^2 + y$ terms. Half of 1 is 0.5, so rewrite them as $(y + 0.5)^2$ and add $0.5^2 = 0.25$ to both sides.
- $x^2 + 6x + y^2 + y = 12.25$ Use $(x - h)^2 + (y - k)^2 = r^2$ to find that the $r^2 = 12.25 = \frac{49}{4}$ and $r = \frac{7}{2} = 3.5$. (Of course, if this is in the calculator allowed section, just throw the $\sqrt{12.25}$ into your calculator. But, for the no calculator section, your best bet for square roots or squares of decimals is to use fractions instead.)

1.13 Chapter 14 Fractions and Probability Answers

Explanations for Chapter 14 Drill 1 - Parts of a Total

- $\frac{20}{106}$ There are 106 total customers. The customers who ordered drip coffee are on the top row: the ones who ordered a baked good are on the left column. The ones who did both are on the top left cell, 20 customers out of 106.
- $\frac{18}{106}$ There are 106 total customers we're randomly choosing from. The customers who ordered an espresso drink are on the bottom row: the ones who did not order a baked good are on the right row. The ones who did both are on the bottom right cell, 18 customers out of 106.
- $\frac{52}{106}$ There are 106 total customers. The ones who ordered a baked good are in the left column, which (luckily for us) is already added up in the "total," on the bottom left: 52 customers out of 106.
- $\frac{50}{106}$ There are 106 total customers we're randomly choosing from. Since everyone either ordered a drip coffee or an espresso drink, the customers who did not order a drip coffee must have ordered an espresso drink: 50 total customers out of 106.
- $\frac{68}{106}$ There are 106 total customers we're randomly choosing from. We want to know the ones who ordered a drip coffee and no baked good (36 customers) *or* an espresso and a baked good (32 customers), for a total of 68 customers out of 106 total.

Explanations for Chapter 14 Drill 2 - Subsets and Sample Spaces

- $\frac{9}{12}$ We're only looking at the small dogs, so the sample space is 12 total dogs. Out of those, 9 of them got shampooed.
 - $\frac{3}{15}$ The sample space is the dogs that did not get shampooed: 15 total dogs. Out of those, we want to know the probability that one of those unshampooed dogs was a small dog: 3 dogs.
 - $\frac{6}{18}$ We're looking for a proportion of the medium or large dogs: that's a combined sample space of $7 + 11 = 18$ dogs. Out of those, 2 medium dogs and 4 large dogs were shampooed: $2 + 4 = 6$.
 - $\frac{7}{15}$ The sample space is the dogs that did not get shampooed: 15 total dogs. Out of those, 7 of them were large dogs that did not get shampooed.
 - $\frac{13}{23}$ The sample space is the dogs that are not medium dogs: the 12 small dogs or the 11 large dogs, for a sample space of $12 + 11 = 23$ dogs. Out of those, we're hoping to randomly choose a squeaky clean shampooed dog: either one of the 9 small shampooed dogs or one of the 4 large shampooed dogs. Any of those 13 dogs are good. (Actually, all dogs are good dogs.)
 - $\frac{2}{96}$ The sample space is the visitors to Park A: 96 total visitors. Out of those, 2 of them were over 65 years old.
 - $\frac{32}{50}$ The sample space is the visitors under 18: 50 total visitors. Out of those visitors under 18, 32 of them went to Park C.
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8. $\frac{26}{102}$ We're randomly choosing one of the visitors who is over 50, so the sample space includes everyone 51-65 years old (43 visitors) and anyone over 65 years old (59 visitors), for a total sample space of 102 visitors. Out of those, 23 visitors to Park C were 51-65 years old, and 3 visitors to Park C were over 65 years old: 26 visitors to Park C were over 50 years old.
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9. $\frac{18}{186}$ "Given that they were at Park A or Park B" means that it is a given that they are at either Park A (96 visitors) or Park B (90 visitors). Those $96 + 90 = 186$ visitors make up our sample space. Out of those, we want the visitors to Park A or Park B who were under 18: 14 visitors to Park A were under 18, and 4 visitors to Park B were under 18: $14 + 4 = 18$.
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10. $\frac{30}{127}$ The sample space is all the visitors aged 36 to 65. That includes the 84 visitors who were aged 36-50 and the 43 visitors who were aged 51-65 for a total sample space of 127 visitors. Out of those, we want the ones who visited Park B: 18 visitors aged 36-50, and 12 visitors 51-65: $18 + 12 = 30$ visitors.

1.14 Chapter 15 Everything Else Answers

Explanations for Chapter 15 Practice Drill 1 - Area, Perimeter, and Volume

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| 1. | $32 = (2w)w$
$32 = 2w^2$
$16 = w^2$
$4 = w$ | <p>Plug in what we know into the area formula for a rectangle: $A = \ell w$. The area is 32, so $A = 32$, and the length is twice the width, so $\ell = 2w$.</p> <p>Combine like terms.</p> <p>Divide both sides by 2.</p> <p>Take the square root of both sides.</p> |
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| 2. | $A = \frac{1}{2} \left(\frac{1}{2}h \right) h$
$A = \frac{1}{4}h^2$ | <p>Start with the formula for the area of a triangle: $A = \frac{1}{2}bh$. We know the base is one-half the height, so $b = \frac{1}{2}h$. Put that into the formula for b.</p> <p>Combine like terms.</p> |
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| 3. | $120 = 5(6)(a)$
$120 = 30a$
$4 = a$ | <p>Put what we know into the volume formula, $V = \ell wh$.</p> <p>Combine like terms.</p> <p>Divide both sides by 30.</p> |
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| 4. | $16\pi = \pi r^2(4)$
$4 = r^2$
$2 = r$
$4 = d$ | <p>Plug in what you know to the volume formula for a cylinder: $V = \pi r^2 h$.</p> <p>Divide both sides by 4π.</p> <p>Take the square root of both sides to find the radius.</p> <p>The diameter is twice the radius.</p> |
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| 5. | $V = \pi r^2(2r + 15)$
$V = 2\pi r^3 + 15\pi r^2$ | <p>The volume of a cylinder is given by $V = \pi r^2 h$. The height, h, is “15 inches longer than the inner width.” The inner width of the pipe is the diameter, $2r$, so 15 inches longer than the inner width is $2r + 15$. Put that in for h.</p> <p>Distribute.</p> |
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| 6. | $\frac{\pi}{4} = \frac{1}{3}\pi r^2(12)$
$\frac{\pi}{4} = 4\pi r^2$
$\frac{\pi}{16} = \pi r^2$
$\frac{1}{16} = r^2$
$\frac{1}{4} = r$ | <p>Put everything we’re given into the formula for the volume of a cone: $V = \frac{1}{3}\pi r^2 h$.</p> <p>Combine like terms on the right side. $\frac{1}{3}(12) = 4$.</p> <p>Divide both sides by 4.</p> <p>Divide both sides by π.</p> <p>Take the square root of both sides.</p> |
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- 7.
- $$50 = \pi r^2 h$$
- Normally I would say let's make up a value for r and h to give us a volume of 50, but since there's no π on the left side, that's going to be tougher than usual. We could make $h = \frac{1}{\pi}$ or something, but let's hold off on that. For now, I've just written the volume formula for cylinder A and put in what we know: it has a volume of 50.
- $$V_B = \pi(3r)^2 \left(\frac{1}{2}h\right)$$
- For cylinder B, we can use the same cylinder volume formula, $V = \pi r^2 h$, and put in what we know about the radius (it's "3 times the radius of cylinder A," or $3r$) and the height (it's "half the height of cylinder A," or $\frac{1}{2}h$).
- $$V_B = \pi 9r^2 \left(\frac{1}{2}h\right)$$
- $$(3r)^2 = 9r^2.$$
- $$V_B = \frac{9}{2}\pi r^2 h$$
- Combine like terms. I'm bringing the 9 and the $\frac{1}{2}$ out front, so we can isolate $\pi r^2 h$.
- $$V_B = \frac{9}{2}50$$
- We know that $\pi r^2 h$ is the volume of cylinder A, which is 50. So replace $\pi r^2 h$ with 50 in the equation.
- $$V_B = 225$$
- Multiply out $\frac{9}{2}(50) = 225$.

- 8.
- $$B = \frac{1}{2}10(10)$$
- The volume of a right triangular prism is the area of the base, B , times the height, h . In this case, the area of the base is the area of a triangle, $A = \frac{1}{2}bh_T$. (Note: that h_T is a different h than the height of the cylinder. It's the height of the triangle. A little confusing, I know. I made it h_T to specify that it's the height of the triangle.) Since this triangle has sides of 10, 10, and $10\sqrt{2}$, it's a $45^\circ - 45^\circ - 90^\circ$ triangle (as discussed on page ??), and the base and height are both the same, so $b = 10$ and $h_T = 10$. (The other length, $10\sqrt{2}$, is the hypotenuse.)
- $$B = 50$$
- Combine like terms. Now we know the area of the triangular base is 50.
- $$V = 50(5) = 250$$
- The volume is $V = Bh$. We've now found B and the problem says the height is 5.

- 9.
- $$\frac{4}{3}\pi r^3 = \pi r_c^2 h$$
- The question tells us that the volume of the sphere is equal to the volume of the cylinder. (I made the radius for the cylinder r_c , to make it clear that it's different from the radius of the sphere, r .)
- $$\frac{4}{3}\pi r^3 = \pi(2r)^2 9$$
- The radius of the cylinder is twice the radius of the sphere, so $r_c = 2r$. Replace the radius of the cylinder in the equation with $2r$, twice the radius of the sphere. Also, the height of the cylinder is 9, so put $h = 9$ in there as well.
- $$\frac{4}{3}\pi r^3 = \pi 4r^2 9$$
- $$(2r)^2 = 2^2 r^2 = 4r^2.$$
- $$\frac{4}{3}\pi r^3 = 36\pi r^2$$
- Combine like terms on the right.
- $$\frac{4}{3}r^3 = 36r^2$$
- Divide both sides by π .
- $$\frac{4}{3}r = 36$$
- Divide both sides by r^2 .
- $$r = 27$$
- Multiply both sides by $\frac{3}{4}$ to get rid of the fraction on the left.

- 10.
- | | |
|---|---|
| $64\pi = \pi r^2$ | Find the radius of the circle first, using $A = \pi r^2$. |
| $64 = r^2$ | Divide both sides by π . |
| $8 = r$ | Take the square root of both sides. This radius is also the diagonal of the square. |
| $8 = x\sqrt{2}$ | For any square, the diagonal is the hypotenuse of a right triangle: specifically, a $45^\circ - 45^\circ - 90^\circ$ triangle (as discussed on page ??), so the radius is the same as the $x\sqrt{2}$ side. |
| $\frac{8}{\sqrt{2}} = x$ | Divide both sides by $\sqrt{2}$. Each side of the square is $\frac{8}{\sqrt{2}}$. |
| $A = \left(\frac{8}{\sqrt{2}}\right)^2$ | Find the area of the square. $A = s^2$. |
| $A = \frac{64}{2}$ | Square the top and the bottom. |
| $A = 32$ | Reduce the fraction. |

Explanations for Chapter 15 Practice Drill 2 - Absolute Values

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| 1. | $(x - 2) = 8$ or $-(x - 2) = 8$ | Either $x - 2$ is positive, or it's negative: either way, it will give us 8. |
| | $x = 10$ | Solve the first equation by adding 2 to both sides, and we have a positive value of x , so we're done. |
| <hr/> | | |
| 2. | $(2x + 7) = 15$ or $-(2x + 7) = 15$ | Either $2x + 7$ is positive, or it's negative: either way, it equals 15. |
| | $2x = 8$ | Solve the first equation by subtracting 7 from both sides. |
| | $x = 4$ | Divide both sides by 2. This is the positive solution for x , so that's the answer. |
| <hr/> | | |
| 3. | $(x - 5) = 7$ or $-(x - 5) = 7$ | Either $x - 5$ is positive, or it's negative. But it always equals 7. |
| | $x = 12$ | For the first equation, add 5 to both sides. One solution to the equation is 12. |
| | $-x + 5 = 7$ | Now solve the other equation. Distribute the -1 to each term in the parentheses. |
| | $-x = 2$ | Subtract 5 from both sides. |
| | $x = -2$ | Multiply both sides by -1 . The solutions for the equation are $x = 12$ or -2 . |
| <hr/> | | |
| 4. | $(x + 6) = 0$ or $-(x + 6) = 0$ | Either everything in the absolute value is positive, or it's negative. |
| | $x = -6$ | Solve the first equation by subtracting 6 from both sides. |
| | $-x - 6 = 0$ | For the second equation, distribute the -1 to each term in the parentheses. |
| | $-x = 6$ | Add 6 to both sides. |
| | $x = -6$ | Multiply both sides by -1 , and we get the exact same answer we already had. Huh. So, it looks like the only answer to this equation is $x = -6$, and the equation has 1 solution. |

$$(x - 9) = -(x + 3)$$

Since we have two absolute value equations, either one of them could be positive or negative. If you solve the equation with both absolute values as positive or both of the absolute values as negative, you'll get nonsense answers, like $0 = 12$. So let's set one of the absolute values to positive and the other to negative. It doesn't matter which is which.

5.

$$x - 9 = -x - 3$$

Distribute the -1 to each term in the parentheses.

$$x = -x + 6$$

Add 9 to both sides.

$$2x = 6$$

Add x to both sides.

$$x = 3$$

Divide both sides by 2.