Contents

11

Exponents, Exponential Growth, and Imaginary Numbers

11.1 What Should I Work On?

Try the following questions, which will help you figure out which portions of this chapter you need to focus on the most.

1

If $a^7 = 2$, where a is a real number, what is the value of a^{21} ?

A) $3\sqrt[7]{2}$

B) $\sqrt[3]{21}$

C) 6

D) 8

2

If $x > 0$ and $y > 0$, which of the following is equivalent to $\sqrt[3]{64x^8y^6}$?

A) $4x^{\frac{8}{3}}y^2$

B) $4x^2y^{\frac{3}{2}}$

C) $8x^{\frac{8}{3}}y^2$

D) $8x^2y^{\frac{3}{2}}$

3

Which of the following complex numbers is equivalent to $5i(8-3i)+2i$? (Note: $i = \sqrt{-1}$)

A) $15+42i$

B) $-15+38i$

C) $40i-17i^3$

D) $-40+17i$

4

Each year, the total revenue for a publishing company increases by 2.1% compared to the previous year. Which type of function best models the annual revenue for the publishing company?

A) Decreasing linear

B) Decreasing exponential

C) Increasing linear

D) Increasing exponential

5

A researcher measures 20 square feet of algae growing on a pond. According to her model, for a certain length of time after this measurement, the surface area covered by algae will triple every day. Which function could be the researcher's model of the surface area, s, covered by the bacteria after t days?

A) $s(t) = 20(3)^t$

B) $s(t) = 20t^3$

C) $s(t) = (20+t)^3$

D) $s(t) = 60^t$

6

$$W(x) = 31.96(1.06)^x$$

A group of researchers is investigating the relationship between weight and length in the spiny angelshark. The function above gives the estimated weight W, in grams, for a spiny angelshark based on its length x, in centimeters. Which of the following is the best interpretation for the number 1.06 in this context?

A) For each increase of 1 gram in weight, the estimated length of the spiny angelshark increases by 1.06 centimeters.

B) For each increase of 1 centimeter in length, the estimated weight of the spiny angelshark increases by 1.06 grams.

C) For each increase of 1 gram in weight, the estimated length of the spiny angelshark increases by 6%.

D) For each increase of 1 centimeter in length, the estimated weight of the spiny angelshark increases by 6%.

7

A planner for a city's mass transit system estimates that, starting from the current year, the population of the city will increase by 20% every 10 years. If the current population of the city is 75,000 people, which of the following functions represents the planner's estimate of the population P of the city in t years?

A) $P(t) = 75{,}000(0.2)^{10t}$

B) $P(t) = 75{,}000(0.2)^{\frac{t}{10}}$

C) $P(t) = 75{,}000(1.2)^{10t}$

D) $P(t) = 75{,}000(1.2)^{\frac{t}{10}}$

8

$$R(t) = 3{,}100(1.08)^t$$

The equation above models the number of rabbits, R, on an island t years after the first measurement. Which of the following models the number of rabbits on the island q quarter years after the first measurement?

A) $R = 3{,}100(1.08)^{4q}$

B) $R = 3{,}100(1.08)^{\frac{q}{4}}$

C) $R = 3{,}100(1.02)^{4q}$

D) $R = 3{,}100(1.02)^{q}$

9

The population of a particular U.S. county decreases over time. The population, P, of the county t years after 1975 is given in the table below.

Years t after 1975	Population P
0	130,000
15	82,323
30	52,131
45	33,012

A statistician models this relationship with the function $P(t) = h \cdot 10^{kt}$. Which of the following are possible values of h and k ?

A) $h = 13{,}000$ and $k = -0.0423$

B) $h = 13{,}000$ and $k = 0.0423$

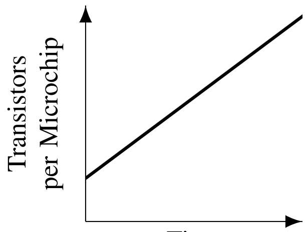
C) $h = 130{,}000$ and $k = -0.0132$

D) $h = 130{,}000$ and $k = 0.0132$

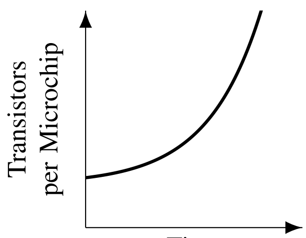
10

In 1975, Gordon Moore predicted that the number of transistors per microchip would double every 2 years. Which of the following graphs best models the number of transistors per microchip according to Moore's prediction?

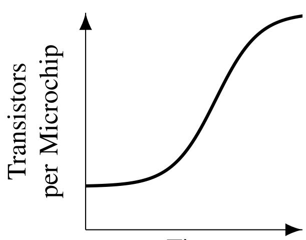
A)



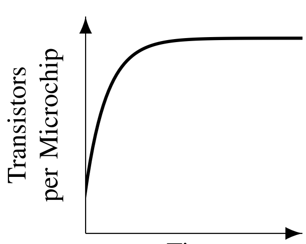
B)



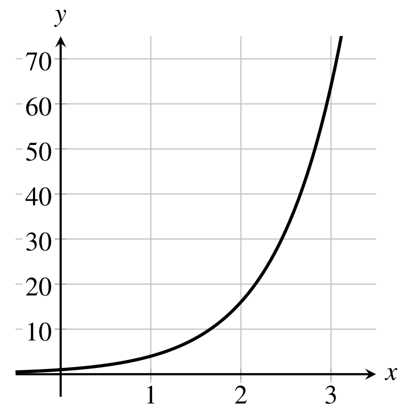
C)



D)



11



The graph above is the exponential function $f(x)$. Which of the following best models the function f?

A) $f(x) = 3^{-x}$

B) $f(x) = 3^{x}$

C) $f(x) = 4^{-x}$

D) $f(x) = 4^{x}$

What Should I Work On? Answers

Question	Answer	If you got this question wrong, go to section...
1	D	Exponents, page 8
2	A	
3	A	Imaginary Numbers, page 12
4	D	Exponential Growth, page 14
5	A	Exponential Function Formula, page 15
6	D	
7	D	Time Intervals, page 18
8	B	
9	C	Using Values, page 20
10	B	Exponential Graphs, page 21
11	D	

Full explanations on page 34 in Chapter 12, but before you go there, read through the section in this chapter that explains how you can solve that type of problem, and then try the question one more time.

11.2 Exponents

An exponent is just a way of writing repeated multiplication. For instance, $5 \times 5 = 5^2$, or $7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^6$.
There aren't many rules you need to know for exponents. Because these are questions that could be answered pretty easily with a calculator, the SAT tends to ask exponent questions in the No Calculator section. You'll just have to use a couple exponent rules to rewrite expressions or solve equations.

Multiplication → Add the exponents	$(x^a) \cdot (x^b) = x^{a+b}$
Division → Subtract the exponents	$\frac{x^a}{x^b} = x^{a-b}$
Power of a Power → Multiply the exponents	$(x^a)^b = x^{ab}$

Since an exponent is just a shorthand way of writing multiplication, it's easy to prove these rules, in case you forget.

$$\begin{aligned}(y^3)(y^2) &= (y \times y \times y)(y \times y)\\ &= y \times y \times y \times y \times y\\ y^{2+3} &= y^5\end{aligned}$$

$$\begin{aligned}\frac{y^5}{y^3} &= \frac{y \times y \times y \times y \times y}{y \times y \times y}\\ &= y \times y\\ y^{5-3} &= y^2\end{aligned}$$

$$\begin{aligned}(y^3)^2 &= (y \times y \times y)^2\\ &= (y \times y \times y)(y \times y \times y)\\ &= y \times y \times y \times y \times y \times y\\ y^{3\times 2} &= y^6\end{aligned}$$

There are two more rules, both of which come from the division rule. The first is that any number raised to the 0th power is 1. You can prove it yourself by dividing any number with an exponent by itself.

$$\begin{aligned}\frac{y^3}{y^3} &= \frac{y \times y \times y}{y \times y \times y}\\ y^{3-3} &= \frac{1}{1}\\ y^0 &= 1\end{aligned}$$

The other rule that arises from the rule for division is the definition of negative exponents: a negative exponent is just a way of showing the reciprocal. You can think of it as the result of dividing, but having more on the bottom of the fraction than on the top.

$$\begin{aligned}\frac{y^2}{y^5} &= \frac{y \times y}{y \times y \times y \times y \times y}\\ y^{2-5} &= \frac{1}{y \times y \times y}\\ y^{-3} &= \frac{1}{y^3}\end{aligned}$$

Any number to the power of 0 is 1.
$x^0 = 1$

A negative exponent is the reciprocal.
$x^{-2} = \frac{1}{x^2}$

Example 1

If $x > 1$ and $(x^5)(x^7) = (x^3)^b$, what is the value of b ?

A) $\frac{3}{35}$

B) 4

C) 9

D) $\frac{35}{3}$

Start with the left side. Since we're multiplying, we're going to add the exponents:

$$\begin{aligned}(x^5)(x^7) &= (x^3)^b \\ x^{5+7} &= (x^3)^b \\ x^{12} &= (x^3)^b\end{aligned}$$

Now work on the right side. We're raising a variable to another power, so we'll multiply the variables.

$$x^{12} = x^{3b}$$

Those two exponents have to be equal to make both sides of the equation equal.

$$\begin{aligned}12 &= 3b \\ 4 &= b\end{aligned}$$

Practice Drill 1 - Exponent Rules

Solve for the value of a. In all questions, assume that x is a positive real number. Answers on page 33, along with full explanations online at *laserfocusprep.com*.

1. If $x^a = (x^5)(x^8)$, what is the value of a ?
2. If $x^a = \frac{x^9}{x}$, what is the value of a ?
3. If $x^a = \left(x^5\right)^8$, what is the value of a ?
4. If $x^a = \frac{(x^7)(x^3)}{x^4}$, what is the value of a ?
5. If $x^a = \left(\frac{(x^6)(x^{10})}{x^4}\right)^3$, what is the value of a ?
6. If $x^a(x^5) = \left(x^9\right)^{12}$, what is the value of a ?
7. If $x^a = \frac{x^8}{x^{-4}}$, what is the value of a ?
8. If $x^a = (x^{-4})(x^3)^5$, what is the value of a ?
9. If $x^a = \left(\frac{x^{-2}}{x^6}\right)\left(x^2\right)^8$, what is the value of a ?
10. If $x^5 = 3$ and $a = x^{15}$, what is the value of a ?

11.2.1 Radicals and Fractional Exponents

The most commonly tested exponent skill on the SAT is rewriting radicals as exponents. Every root can be rewritten as a fractional exponent, where the denominator of the exponent tells us which root we're using. In other words, $\sqrt{x} = x^{\frac{1}{2}}$. A square root is the same as a $\frac{1}{2}$ exponent. For a cube root, $\sqrt[3]{x} = x^{\frac{1}{3}}$. Any root can be rewritten as a fractional exponent.

The denominator of an exponent
gives the root.

$$x^{\frac{1}{a}} = \sqrt[a]{x}$$

All of the exponent rules from the previous section work with fractional exponents!

$$\left(x^{\frac{1}{2}}\right)\left(x^{\frac{1}{2}}\right) = x^{\frac{1}{2}+\frac{1}{2}}$$
$$= x^1$$

That makes sense, because $(\sqrt{x})(\sqrt{x}) = x$.
The numerator of the exponent, the top of the fraction, still tells us the power we're using. We could, for instance, write "2 cubed" as $2^{\frac{3}{1}}$ which is, of course, the same as 2^3. As a fraction, an exponent of $\frac{3}{1}$ means "take the 3rd power and the 1st root." (The 1st root is just the number by itself, so it doesn't do anything.)
That means we can combine powers and roots: $4^{\frac{3}{2}} = \left(\sqrt{4}\right)^3$. By the way, it doesn't matter if you do the square root or the cubed part first: $\left(\sqrt{4}\right)^3 = 2^3 = 8$, and $\sqrt{4^3} = \sqrt{64} = 8$. You'll get the same answer either way.
So each fraction in an exponent gives us the *power* over the *root*. Notice it's in alphabetical order, top to bottom: Power over Root.

A fraction in an exponent is $\frac{power}{root}$

$$x^{\frac{a}{b}} = (\sqrt[b]{x})^a$$

Example 2

$$\left(8x^3y^5\right)^{\frac{1}{3}}$$

Which of the following is equivalent to the expression above?

A) $\frac{8}{3}xy^{\frac{5}{3}}$

B) $2xy^{\frac{5}{3}}$

C) $\sqrt[3]{2xy^5}$

D) $8xy^{\frac{5}{3}}$

Let's apply the $\frac{1}{3}$ power to each part inside the parentheses.

$$\left(8x^3y^5\right)^{\frac{1}{3}}$$
$$8^{\frac{1}{3}}\left(x^3\right)^{\frac{1}{3}}\left(y^5\right)^{\frac{1}{3}}$$
$$8^{\frac{1}{3}}x^{\frac{3}{3}}y^{\frac{5}{3}}$$
$$8^{\frac{1}{3}}x^1y^{\frac{5}{3}}$$

With integers, it's often easiest to rewrite fractional exponents as roots. So $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$, and we have $2xy^{\frac{5}{3}}$, answer choice is (B).

In general, if you have exponents and roots, put everything in exponent form until the end of the question, when you can convert any remaining fractional roots into square roots as needed.

Example 3

If $x = 25$, which of the following is equivalent to $x^{\frac{3}{4}}$?

A) $\sqrt[4]{125}$

B) $3\sqrt[4]{25}$

C) $\sqrt{5}$

D) $5\sqrt{5}$

Since $x = 25$, $x^{\frac{3}{4}} = 25^{\frac{3}{4}}$. 25 isn't a number I really want to cube or take the 4th root of, so let's simplify it a bit. With radicals, it's often easier if we break down anything we can write using exponents, so let's rewrite 25 as 5^2.

$$\begin{aligned}25^{\frac{3}{4}} &= \left(5^2\right)^{\frac{3}{4}}\\ &= 5^{\frac{6}{4}}\\ &= 5^{\frac{3}{2}}\\ &= \left(\sqrt{5}\right)^3\\ &= \left(\sqrt{5}\right)\left(\sqrt{5}\right)\left(\sqrt{5}\right)\\ &= 5\left(\sqrt{5}\right)\end{aligned}$$

Which is the same as answer choice (D). There are several ways of breaking down $5^{\frac{3}{2}}$. I chose to rewrite it as a square root, cubed: a square root because the denominator of the exponent is 2, and cubed because the numerator of the exponent is 3. Once I had a cubed square root I could expand it out and change $\left(\sqrt{5}\right)\left(\sqrt{5}\right) = 5$. However, since $\frac{3}{2} = \frac{2}{2} + \frac{1}{2}$, you could use the rule for multiplying exponents and think of $5^{\frac{3}{2}}$ as $5^{1+\frac{1}{2}} = \left(5^1\right)\left(5^{\frac{1}{2}}\right)$. Or you could find $\sqrt{5^3} = \sqrt{125} = \left(\sqrt{25}\right)\left(\sqrt{5}\right) = 5\sqrt{5}$. Whichever way makes the most sense to you at that point works fine.

Practice Drill 2 - Radicals and Fractional Exponents

For each question, match the expression with the simplified expression from the list below. Multiple questions may match with the same expression. Answers on page 33, along with full explanations online at *laserfocusprep.com*.

A) $\sqrt{x^3}$	B) $\sqrt[3]{x^2}$	C) $\frac{1}{\sqrt{x}}$	D) $\frac{1}{x^2}$	E) $x^{\frac{7}{10}}$	F) $x^3\sqrt{x}$
G) $x^3\sqrt[3]{x}$	H) $x\sqrt[3]{x}$	I) $\sqrt[6]{x^5}$	J) $4x^2$	K) $4x^4$	L) $8x^4$

1. $x^{-\frac{1}{2}}$
2. $x^{\frac{2}{3}}$
3. $x^{\frac{4}{3}}$
4. $\sqrt{x^4(x^3)}$
5. $x^{\frac{1}{5}}\sqrt{x}$
6. $\sqrt[3]{x}\sqrt{x}$
7. $\frac{\sqrt[3]{x^5}}{x}$
8. $\left(16x^8\right)^{\frac{1}{2}}$
9. $\frac{x^{\frac{5}{2}}}{\sqrt[3]{x^9}}$
10. $\sqrt[3]{4x^3}\left(\sqrt[3]{2x}\right)\left(2\sqrt[3]{x^2}\right)$

11.3 Imaginary Numbers

For every square root, we've looked only at the square roots of positive numbers. Those are the square roots that generally make more sense: if a square has an area of 25 inches^2 then each side is $\sqrt{25} = 5$ inches. What's $\sqrt{-25}$, however? As it is, we can't solve it. So, instead we can break it apart into two pieces: $\sqrt{25}\sqrt{-1} = 5\sqrt{-1}$. Now we've taken the negative square root and set it aside. To make things a bit easier, mathematicians gave a name to the $\sqrt{-1}$: they called it i, the imaginary number.
We can add and subtract i just like we can any term, just as we would anything with an x. In the same way that $5x + 3x = 8x$, $5i + 3i = 8i$. Or, just as $9x - 2x = 7x$, $9i - 2i = 7i$.
Many of the questions with imaginary numbers will just ask you to combine terms, in which case you can simplify whatever expression you're given in the same way you would with an expression in terms of x.
A *complex number* is any number that includes i. It's made up of a real part (the numbers we're used to) and an imaginary part (a number with i). So $4 + 5i$ is a complex number, with real part 4 and imaginary part 5. This is only really important for us because the SAT will refer to complex numbers or *the complex number system*, which just means that we have terms that include i. The SAT will also ask about complex numbers by saying things like "if (something) is a complex number of the form $a + bi$, what is the value of a" or b, or ab or $a + b$ or whatever. In that case, just match up the real and imaginary parts: for $5 - 3i = a + bi$, for instance, $a = 5$ and $b = -3$.
There's only one other thing to know about imaginary numbers. Since $i = \sqrt{-1}$, then $i^2 = \left(\sqrt{-1}\right)^2 = -1$. Any time you see i^2, replace it with -1. For anything larger, break it up into as many i^2 terms as possible. For instance, $i^3 = \left(i^2\right)i = (-1)i = -i$,

and $i^4 = \left(i^2\right)\left(i^2\right) = (-1)(-1) = 1$, and so on and so on. The SAT doesn't ask about anything larger than i^2 too often, so don't worry about memorizing the values of i other than i^2, unless, of course, you either really want to or you need to for math class anyway.

Questions about imaginary numbers are nearly always asked in the No Calculator section, because many calculators are programmed to do math with imaginary numbers. Many calculators can find the value of, say, $(3i+2)(5i-1)$ and instantly give you the answer, $-17+7i$. So, to avoid being helpful in any way, the SAT generally has questions involving imaginary numbers only in section 3.

Example 4

$$(8i+7)-(2i^2-3i)$$

Which of the following complex numbers is equal to the expression above, for $i=\sqrt{-1}$?

A) $5+5i$

B) $7+3i$

C) $9+5i$

D) $9+11i$

As is pretty common with parentheses (and as discussed in Chapter 2 on page **??**), parentheses are a common way for the SAT to test whether you remember to distribute with subtraction.

$$(8i+7)-(2i^2-3i)$$
$$(8i+7)+-2i^2-(-3i)$$
$$8i+7-2i^2+3i$$
$$11i+7-2i^2$$

Now use $i^2=-1$

$$11i+7-2(-1)$$
$$11i+7+2$$
$$11i+9$$

Answer choice (D). The answers for complex numbers will always be written in the form $a+bi$, so you may have to rearrange your answer to match the answer that the SAT gives, putting the real term first and the imaginary term second.

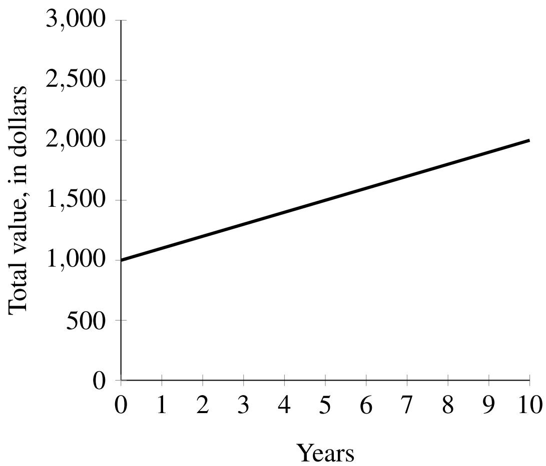
Practice Drill 3 - Imaginary Numbers

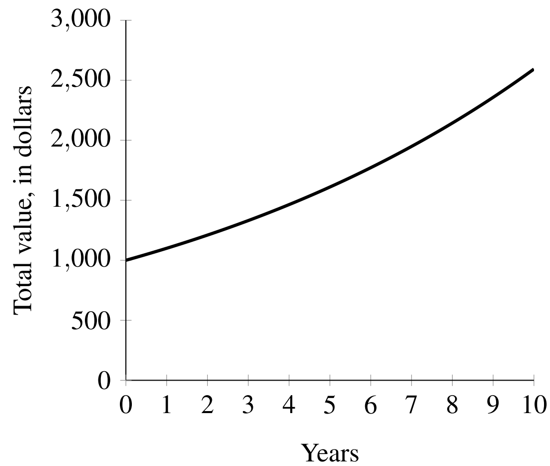
For each question, using $i=\sqrt{-1}$, give your answer in the form $a+bi$. Answers on page 34, along with full explanations online at *laserfocusprep.com*.

1. $(3+12i)+(8+2i)$
2. $(7-8i)+(12+7i)$
3. $(12+4i)-(5-3i)$
4. $(4+i)+(5+i^2)$
5. $(-i)^2-i^2$
6. $8i(3i-2)$
7. $(3+2i)(5+4i)$
8. $(2-5i)(3+6i)$
9. $(1-i)(1+i)$
10. $12i^3+6$

11.4 Exponential Growth

In Chapter 4: Linear Equations, we discussed linear growth and slope. Anything that grows linearly increases by the same amount every time. If I say "I give you $1,000, and every year I will send you another $100," then that is linear growth: every year, you're getting the same amount. If you put all the money I give you in an envelope, then the amount of money in that envelope will increase by the exact same amount every single year: $100.
But let's instead say that you open up a bank account with a very generous bank that agrees to give you 10% interest every year. You deposit $1000 dollars, and after a year the bank gives you 10% of $1000, which is $100. So far, you're earning the same amount in a year as you were when you were putting $100 bills in an envelope that just sat there.
Next year, however, you're going to earn a bit more. Now you have a total of $1,100 in your account, so when the bank gives you that 10% interest, it will give you 10% of $1,100, which is $110. The amount of money you're earning each year in your account is increasing. Next year, it'll be even more: 10% of the $1,210 that's now in your account. The year after that, even more. You're earning interest on the initial amount you deposited *and* you're earning interest on the interest you've earned. Every year, the amount added to the account increases.
Here's what the two situations look like as graphs.





On the left, you started with $1000 and got $100 every year. It's an increasing linear function, a nice straight line. On the right, you also started with $1000, but you earned 10% every year. Notice that it's a curved line, as the amount that you earn increases every year.
That's an exponential function. In this case, this is an *increasing exponential function*.

Example 5

Rock's law predicts that the cost of a factory to make semiconductor chips doubles every four years. If Rock's law is modeled by a function that gives the predicted cost R of a semiconductor factory where t is the time, in years, which of the following describes the function $R(t)$?

A) Increasing linear

B) Increasing exponential

C) Decreasing linear

D) Decreasing exponential

First off, the price of the factory is increasing, so eliminate (C) and (D).
With this sort of question, we need to figure out whether the value is changing by the *same amount* each year, or by an *increasing amount* each year. Every four years, the cost of the factory doubles. A factory that cost $5 million would cost $10 million after 4 years, or $20 million after 8 years. The cost of the factory isn't increasing linearly, it's increasing by more each time. It's an increasing exponential function, answer choice (B).

Practice Drill 4 - Linear and Exponential Functions

For each question, answer what type of function it is from the options A - D below. Answers on page 34, along with full explanations online at *laserfocusprep.com*.

A) Increasing linear

B) Decreasing linear

C) Increasing exponential

D) Decreasing exponential

1. Every month 20 more books are added to the library's collection.
2. Every month a website has twice as many visitors as it did the previous month.
3. 12 new members join the jogging club every year.
4. The population of a country decreases by 9% per year.
5. Every year the price of a certain computer goes down by 20%.
6. The price of a magazine subscription increases by $10 per year.
7. The speed of a truck decreases at a constant rate of 3 mph per second.
8. Every decade the size of a wetland decreases by half.
9. The value of a company has decreased by 1.2 million dollars each decade.
10. The number of voles in a protected habitat triples each decade.

11.4.1 Exponential Function Formula

One common type of exponential function question is a doubling question. If an amount doubles, it's not increasing by the same amount each time: it's increasing by twice as much as it did last time.

Example 6

A computer program is modeling the behavior of bacteria. The program starts with one simulated bacteria and then doubles the number of bacteria every 15 minutes. According to the program, how many total bacteria are there after 2 hours?

A) 4

B) 64

C) 128

D) 256

For this question, it's easiest to just write out each change in a table. We know that after 15 minutes there will be 2 bacteria, after 30 minutes there will be 4, and so on. We want to know the number in 2 hours, which is the same as 120 minutes.

Minutes	Bacteria
0	1
15	2
30	4
45	8
60	16
75	32
90	64
105	128
120	256

So after 2 hours (120 minutes), there will be 256 total bacteria.

There's another way of solving this: we start off with 1 bacteria. We're going to multiply that bacteria by 2 after 15 minutes, by 2 again after the next 15 minutes, and so on. Overall, that single bacteria will double 8 times: $1(2)(2)(2)(2)(2)(2)(2)(2) = 1(2)^8 = 1(256) = 256$.

The initial number of bacteria was 1, and then we doubled it 8 times. $1 \times (2)^8$. A function with an exponent. An exponential function, you could say. Interesting.

Let's return to the example of interest in a bank. Let's say you start with \$1000 again, just like in the last section, and this (more reasonable) bank gives you 3% interest every year. As discussed in Chapter 7: Percents, we can find how much is in the account after one year by multiplying the starting amount, \$1000, by 1.03. (The 1.03 represents that we will have 100% of what we had the last year, plus another 3%, for a total of 103%, which is 1.03.) We could calculate that exact amount, but I'm going to leave it as 1000(1.03) for now.

To find out how much we'd earn in the second year, we'd take all the money we earned in the first year, a total of 1000(1.03) dollars, and multiply it by 1.03 again, to represent another 3% interest earned: 1000(1.03)(1.03).

Year	Total value
0	\$1000
1	\$1000(1.03)
2	\$1000(1.03)(1.03)
3	\$1000(1.03)(1.03)(1.03)
4	\$1000(1.03)(1.03)(1.03)(1.03)

Every time another year passes, we multiply by 1.03 another time. If 8 years have passed? We would multiply 1000 by 1.03 a total of 8 times. We could write that using exponents, of course: $1000(1.08)^8$. Here's that table above, but this time using exponents.

Year	Total value
0	$\$1000(1.03)^0$
1	$\$1000(1.03)^1$
2	$\$1000(1.03)^2$
3	$\$1000(1.03)^3$
4	$\$1000(1.03)^4$

And so on and so on. After 50 years? That bank account will have $1000(1.03)^{50}$ dollars in it. This is, as you've probably guessed, why these functions are called *exponential functions*. They're functions, they have exponents. You get it.

The formula for exponential functions will always be in this same form. We'll have an initial amount, which in our case was 1000. That will be multiplied by the *rate of change*, which in our case was 1.03, to represent that the amount increased by 3%. We're multiplying by the 1.03, our rate of change, however many times we've increased the value. For our bank, it increases once per year, so however many years have passed, that's the exponent.

> The formula for an exponential function is
> $y = a(b)^x$
>
> For most word problems, this means that:
> y is the final amount
> a is the initial amount
> x is the number of times the value has changed
> b is the rate or common ratio between terms
> For percent increase or decrease, the rate is $1 \pm \frac{\text{percent}}{100}$

Sometimes, instead of x, we'll have t for time, or some other variable, but it's basically the same. For the bacteria problem in Example 6, the initial amount a was 1, our rate b was 2, and x, the number of times it increased, was 8. For the bank example above, the initial amount a was 1000, the rate b was 1.03, and x represented however many years have passed.

Example 7

Since 1970, the population of Argentina has grown by an average rate of 1.4% The population of Argentina in 1980 was approximately 28 million people. Which of the following functions represents the population of Argentina P, in millions of people, t years since 1980? (1 million = 1,000,000)

A) $P(t) = 28(1.4)^t$

B) $P(t) = 28(1.14)^t$

C) $P(t) = 28(1.014)^t$

D) $P(t) = 1.014t + 28$

This is an exponential growth problem: the population of Argentina doesn't increase by the same number of people each year, it increases by 1.4% over the previous year. Eliminate (D).
Using the formula for an exponential function, $y = ab^x$, let's fill in what we know. The initial amount is 28 million people, since we're starting our function at 1980. So $a = 28$, since the P is in terms of millions of people.
The rate is 1.4%. That means each year the population is 100% + 1.4% = 101.4% larger than it was the previous year. $101.4\% = 1.014 = b$, and the answer is (C).

In Chapter 4: Linear Equations, we often found the slope m and the y-intercept b: with those two variables we could write a $y = mx + b$ equation. For exponential functions, we'll just need to find the initial amount a and the rate b.
The initial value will normally be given in the question. The function equals a when $x = 0$. (Notice that since anything to the 0th power is 1, $ab^0 = a(1) = a$, the initial amount.)
For anything that doubles, $b = 2$. For anything that triples, $b = 3$. If it goes down by half, then $b = \frac{1}{2}$.
For exponential functions with percents, remember that the rate will be either above or below 1, because 100% as a decimal is 1.00, more commonly known as 1. An increase of 2% gives $b = 1.02$, and a decrease of 2% gives $b = 0.98$.

Practice Drill 5 - Exponential Functions Formula

For each question, write an equation of the form $y = ab^x$ using the information given. Answers on page 34, along with full explanations online at *laserfocusprep.com*.

1. An initial population of 30 bacteria doubles every single hour. What is the population y of the bacteria after x hours?
2. The price of a particular bicycle declines by 5% every year. If the initial cost of the bicycle was \$350, what is the price y of the bicycle after x years?
3. The value of a company increased 3% per year since 2018, when it was valued at \$34.9 million. What is the value y, in millions, of the company x years after 2018?
4. An invasive species of fish were accidentally released into a river in 1990. Initially, only 12 fish were released, but the number of fish in the river has tripled every year since. What is the number y of fish x years after 1990?
5. A liquid meal is digested by the stomach at a rate of 80% of per hour. How many ounces y of a 12 ounce liquid meal remain after x hours?
6. The annual salary of an employee starts at \$65,000 and increases by 6% each year. What is the annual salary y of an employee after x years?
7. For the first week of March, every day there were half as many boxes in the warehouse as there were the day before. If on March 1st there were 1,664 boxes, what is the number of boxes y in the warehouse x days after March 1st, where $x < 8$?
8. A mutual fund has increased in value by 2.3% every year since 2010. If in 2010 each share was worth \$23, what is the value y of a share of the mutual fund x years after 2010?

9. For an average patient, the amount of a certain medication remaining in their system goes down by a half every week. If an average patient receives a single dose of 4 milligrams of the medication, what is the amount y of the medication remaining in the patient's system after x weeks?
10. As height increases, the pressure decreases. At sea level, the pressure in millimeters of mercury (mmHg) is 760. For each increase in height x of 1 meter, the pressure y in millimeters of mercury decreases by 0.012%. What is the pressure y, in mmHg at a height of x meters?

11.4.2 Time Intervals

Our bank isn't doing too well, and they've decided to only start paying us interest every other year. We could take our money elsewhere, but, well, we love this bank, foolishly, and so we're going to see what happens with our money. Let's start with \$1000 and look at what happens when we get 3% interest every other year.

After one year, well, our bank hasn't done anything. Let's skip that one. But after 2 years, we finally get that 3% interest, and the total amount in our account is $1000(1.03)^1$. When a total of 4 years have passed, the total amount in our account is $1000(1.03)^2$. Here is a table with the first 8 years.

Year	Total value
0	$\$1000(1.03)^0$
2	$\$1000(1.03)^1$
4	$\$1000(1.03)^2$
6	$\$1000(1.03)^3$
8	$\$1000(1.03)^4$

Notice that in each column, our exponent isn't the year anymore. The exponent is *half* of the number of years that have passed. We could write this as $y = 1000(1.03)^{\frac{x}{2}}$. Most of this equation is the same as the one in the previous section, but we've had to *slow down* how often our bank pays interest, so we're dividing the x value, time, by 2. It's only when $x = 2$ and the exponent is $\frac{2}{2} = 1$ that we earn 3%. If we only earned interest every 7 years, then the equation would be $y = 1000(1.03)^{\frac{x}{7}}$ (A truly terrible savings account.)

Wait. I just got news that our bank is suddenly doing way better, and they want to make it up to me, their only remaining customer, by giving me 3% interest two times a year. Every six months ($\frac{1}{2}$ of a year) we'll earn 3% interest. Let's see it in a table again.

Year	Total value
0	$\$1000(1.03)^0$
1	$\$1000(1.03)^2$
2	$\$1000(1.03)^4$
3	$\$1000(1.03)^6$
4	$\$1000(1.03)^8$

This time the exponent is *twice* the number of years that have passed. As an equation, it would be $y = 1000(1.03)^{2x}$. We had to *speed up* the number of times that we earned interest, so we multiplied the exponent by 2.

Let's say we start with 50 bacteria in a petri dish, and every hour the number of bacteria doubles. The formula, just like in the previous section, would be $y = 50(2)^x$ However, if the bacteria population doubled 3 times per hour, then the equation would be $y = 50(2)^{3x}$. If it doubled 4 times an hour? $y = 50(2)^{4x}$ If the bacteria grew more slowly, and only doubled once every 3 hours, then the equation would be $y = 50(2)^{\frac{x}{3}}$. If it grew even more slowly, and only doubled every 10 hours? Then the equation would be $y = 50(2)^{\frac{x}{10}}$.

Notice that when anything changed multiple times per hour, we multiplied the exponent. When it happened once per hour, we divided in the exponent. We can write exponent functions just like we did in the previous section, filling in a, the initial amount, and b, the rate, and then change the exponent as needed.

> If an exponential function changes *multiple* times, then *multiply* the exponent.
> $y = ab^{nx}$ is a function that grows n times per unit x
>
> If an exponential function changes *once every* so often, then *divide* the exponent.
> $y = a(b)^{\frac{x}{n}}$ is a function that grows once every n units of time.

If you're not sure, think about what you have to plug in for x to make the exponent 1. Take the equation $y = 30(1.2)^{\frac{x}{20}}$, where y is the number of flies in an experiment and x is the number of days that have passed. How often does the number of flies grow by 20%? Let's try to make the exponent 1. To make $\frac{x}{20} = 1$, x needs to equal 20, which gives us $y = 30(1.2)^{\frac{20}{20}} = 30(1.2)^1$ So the number of flies only grows by 20% when $x = 20$: once every 20 days.
Now let's look at an equation such as $y = 10(2)^{3x}$, where y is the number of Scotch broom plants in a forest and x is the number of years. What do we have to do to make the exponent 1? To make $3x = 1$, we need x to equal $\frac{1}{3}$, so we have $y = 10(2)^{3\frac{1}{3}} = 10(2)^1$. So the number of Scotch broom plants doubles after $\frac{1}{3}$ of a year, or we can say it doubles 3 times every year.
If you've studied the half-life formula in science, this formula may seem familiar. For a half-life, we'll often have an equation of the form $y = a\left(\frac{1}{2}\right)^{\frac{t}{half-life}}$, because the amount of radioactive material decreases by half for every half-life. (Carbon-14, for instance, has a half-life of 5730 years, so the equation to find the amount of remaining Carbon-14 from an initial sample of 12 grams would be $C(t) = 12\left(\frac{1}{2}\right)^{\frac{t}{5730}}$.

Example 8

The number of zebra mussels is modeled by the function $M(t) = 5(2)^{\frac{t}{3}}$, where $M(t)$ is the number of zebra mussels and t is the time, in days, since the zebra mussels were first introduced into a waterway. What is the best interpretation of $\frac{t}{3}$ in this context?

A) The number of zebra mussels doubled 3 times a day.

B) The number of zebra mussels doubled every 3 days.

C) The number of zebra mussels doubled t times.

D) The number of zebra mussels increased by 2 every $\frac{t}{3}$ days.

For the function $M(t) = 5(2)^{\frac{t}{3}}$, the initial value is 5, and the population doubles. Since the exponent is a fraction, it doubles once every 3 days, answer choice (B).
We can also ask ourselves what value of t gives us an exponent of 1. $\frac{t}{3} = 1$ if $t = 3$, so when 3 days have passed the number of zebra mussels doubles, answer choice (B).

Practice Drill 6 - Time Intervals

For questions 1-5, identify how often the exponential function grows or shrinks by the stated amount. For questions 6-10, write an exponential function of the form $y = ab^{nx}$ or $y = ab^{\frac{x}{n}}$ using the information provided. Answers on page 34, along with full explanations online at *laserfocusprep.com*.

1. For $y = 32(1.18)^{3x}$, where y is value after x days, how often does the value increase by 18%?
2. For $P(t) = 123,000(1.03)^{2t}$, where P is the population t years after 1990, how often does the population increase by 3%?
3. For $F(t) = 142(0.93)^{\frac{t}{5}}$, where F is the temperature in degrees Fahrenheit after t minutes, how often does the temperature decrease by 7%?
4. For $y = 8(3)^{12x}$, where y is the number of organisms in an environment after x years, how often does the number of organisms triple?
5. For $M(t) = 129\left(\frac{1}{2}\right)^{\frac{t}{898}}$ where M is the mass remaining of a radioactive substance and t is the time in years, how often does the amount of radioactive substance decrease by a half?

6. An initial population of 18 goldfish in a pond doubles every 7 years.
7. A sample of 40 bacteria triples 4 times every hour.
8. A stock with a value of \$29 is increasing by 11% once every 2 hours.
9. The value of a house that initially cost \$650,000 decreases by 0.5% 3 times a year.
10. 4 times every hour, a sample of 9 grams of a radioactive substance decays by $\frac{1}{2}$.

Occasionally you'll have to convert units of time for an exponential function. Normally the SAT keeps these units pretty simple, and the only ones you'll need to have memorized are the time units mentioned in Chapter 6: Ratios and Rates, page **??**, which were just the number of hours in a day, the number of minutes in an hour, and the number of seconds in a minute.

It's often easiest to solve these questions by setting the exponent to one time unit (1 day, for instance), and seeing what would be the same in the new function (24 hours, for instance).

Example 9

$$S(t) = 145.2(0.68)^t$$

During a single day of trading, a single stock continually lost value. The value $S(t)$ in dollars of the stock t hours after the market opened is modeled in the equation above. Which of the following functions represents the value of the stock m minutes after the stock opened?

A) $S(m) = 145.2(0.68)^{60m}$

B) $S(m) = 145.2(0.68)^{\frac{m}{60}}$

C) $S(m) = 145.2(0.68)^{\frac{60}{m}}$

D) $S(m) = 145.2(0.68)^{60+m}$

Our original equation is in hours. Let's say that $t = 1$. In that case, our equation is $S(1) = 145.2(0.68)^1$, which represents a single hour of the stock losing value. How do we convert that to minutes? There are 60 minutes in an hour, so if we plug in $m = 60$, we should get the exact same value, $145.2(0.68)^1$, as we did when we plugged in an hour into the original equation.

Answer choice (A) gives us $S(60) = 145.2(0.68)^{60(60)}$, which is wrong. Answer choice (B) gives $S(60) = 145.2(0.68)^{\frac{60}{60}} = 145.2(0.68)^1$ so that works! Let's check the other answers just to make sure: any time you try out your own number for a variable, you should always check all 4 answer choices.

Answer choice (C) gives us $S(60) = 145.2(0.68)^{\frac{60}{60}} = 145.2(0.68)^1$, which also works. (D) is $S(60) = 145.2(0.68)^{60+60}$ which is wrong, so eliminate (D).

We're down to (B) and (C). What if we had used 2 hours? In that case, the value of the stock is $S(2) = 145.2(0.68)^2$. That value should be the same when we convert 2 hours to 120 minutes. Try $m = 120$ on the remaining two answers. Answer choice (B) gives $S(120) = 145.2(0.68)^{\frac{120}{60}} = 145.2(0.68)^2$ so, once again, that works! Answer choice (C) gives $S(120) = 145.2(0.68)^{\frac{60}{120}} = 145.2(0.68)^{\frac{1}{2}}$, so eliminate (C) and the answer choice is (B).

I started with trying out $t = 1$ because it's a bit easier to understand. Most of the time, you'll eliminate 3 answers on your first try and be done. However, if you have answers left, try another number. Or, if the conversions make sense, try out $t = 2$ (or any value that makes the conversions easy) on your first try. It will occasionally save you a step.

There's another way we could solve this: rewrite the problem in terms of minutes. For the current $S(t)$ equation, the price of the stock decreases by 32% every hour, starting at 145.2 dollars. If it is decreasing by 32% per hour, it's decreasing by 32% every 60 minutes. Now we can rewrite the equation using the new time interval. Since it happens "once per 60 minutes," we'll divide the exponent by 60, answer choice (B).

11.4.3 Using Values

Now that we know all the pieces of an exponential function, we can use a function, or write a function, in nearly the same way we're used to with linear equations, as discussed in Chapter 4: Linear Equations. We can insert values that we're given (in the same way we used x- and y-values in linear equations) and we can find any unknown constants in our equations (similar to how we could find b in the $y = mx + b$ equation by plugging in a point and then solving for b).

If you're not given an equation, you may need to write one yourself, in the form $y = ab^x$. (There is a chance they could make you write an exponential function with a different time interval, like we covered in the last section, but these questions are already fairly tough, so the SAT tends to avoid adding too much more to them.)

Once you have an equation, either from the problem or that you wrote yourself, plug in the x and y values you're given.

Example 1

x	$f(x)$
0	16
1	20
2	25

The table above gives some values for a function $f(x)$. If the function is defined by $f(x) = ab^x$, where a and b are constants, what is the value of b?

There are two ways to solve this question. First off, we could just plug in values and solve for each constant. Starting with $f(0) = 16$, we can use that in $f(x) = ab^x$.

$$f(0) = ab^0$$
$$16 = a(1)$$

Now we know that $a = 16$, and so our function is $f(x) = 16b^x$. (You may have already noticed that $f(0)$ is our starting point, the initial amount, which is a, in which case you could skip the above steps and get right to $a = 16$.)

We still need to find b. Let's plug in one of the other points. I'm going to use $f(1) = 20$, because exponents of 1 are easier to deal with.

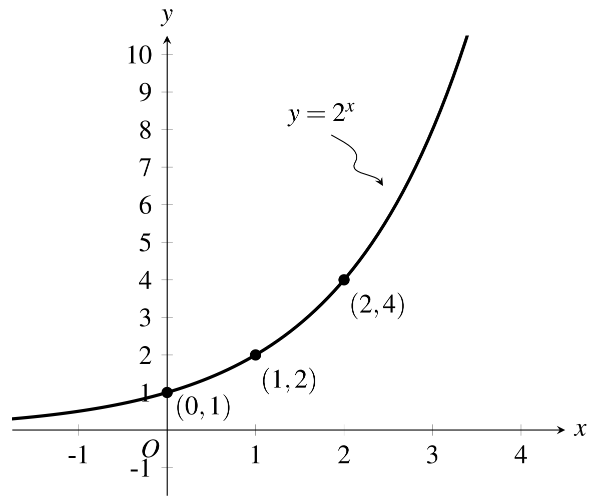
$$f(1) = 16b^1$$
$$20 = 16(b)$$
$$\frac{20}{16} = b$$
$$\frac{5}{4} = b = 1.25$$

There's one other way we could solve this problem. b is our rate, the ratio of each term to the next. You could either calculate the increase between terms (25%, which gives a rate of 125%, so $b = 1.25$), or just take the ratio of any two consecutive terms: $\frac{20}{16} = \frac{25}{20} = \frac{5}{4} = 1.25$.

11.5 Exponential Graphs

Since an exponential function is a function, we can graph it. And, since it's a function, we can transform it just as we did in Chapter 12: Functions and Polynomials, in the Function Transformations section on page **??**, moving the graph of the function left with $f(x+h)$, right with $f(x-h)$, up with $f(x)+k$, or down with $f(x)-k$.

Let's look at a simple exponential function: the doubling function, $y = 2^x$. From the formula, we know that $a = 1$ and $b = 2$, so the initial value is 1 and it doubles every time x increases by 1. Here it is as a graph.

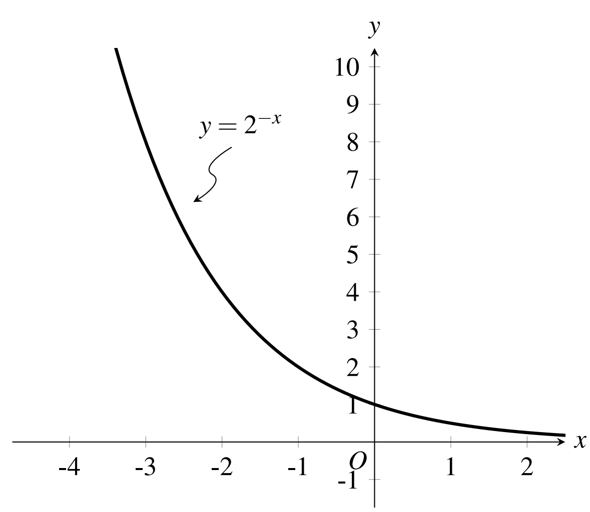


Notice how it goes up faster and faster as you head to the right? That's the usual shape of a positive exponential graph.

The easiest point to find on a graph is the y-intercept, so it's a great place to start with any question about graphs. Here, the y-intercept is our starting value, $a = 1$. As always with the y-intercept, we can find it from the equation by plugging in $x = 0$, to get $y = 2^0 = 1$.

The next point that's easy to check is $x = 1$. For our graph, it's $y = 2^1$, so we know the point (1,2) will be on the graph, which it is.

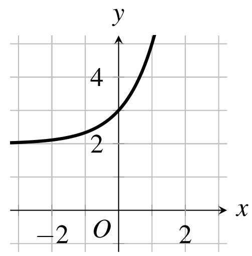
The other type of exponential graph you should know is a *decreasing exponential*. Remember how negative exponents meant that we put everything in the denominator of the fraction? That means that y gets smaller as x gets larger.

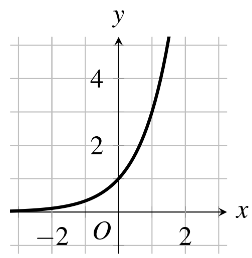


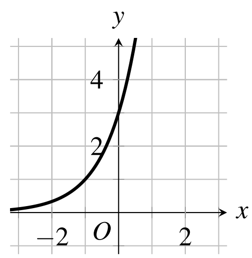
The graph of $y = 2^{-x}$, by the way, is the exact same as the graph of $y = \left(\frac{1}{2}\right)^x$ above, since $2^{-1} = \frac{1}{2}$.

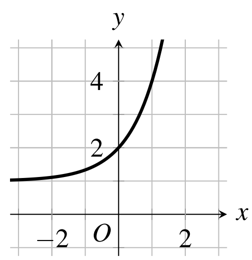
Example 11

Which of the following is the graph in the xy-plane of the equation $y = 3^{x+1}$?

A) 

B) 

C) 

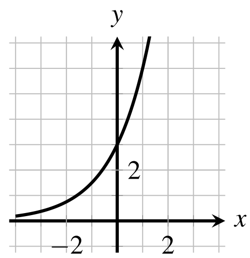
D) 

Start by finding the y-intercept by plugging in $x = 0$. In that case, $y = 3^{0+1} = 3^1 = 3$, so we know the point (0, 3) will be on the graph. Eliminate answer choices (B) and (D).
Now try another point. I'm going to try $x = -1$ because it's easy to see the y-coordinate at $x = -1$ on the graphs for (A) and (C). $y = 3^{-1+1} = 3^0 = 1$, and we know the point $(-1, 1)$ will be on the graph: eliminate answer choice (A), leaving only (C).

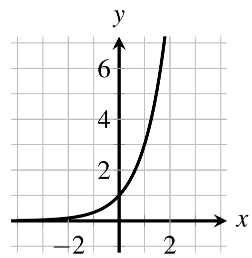
Practice Drill 7 - Exponential Graphs

Identify which of the following could be the graph for each equation given. Each equation matches with exactly one graph. Answers on page 34, along with full explanations online at *laserfocusprep.com*.

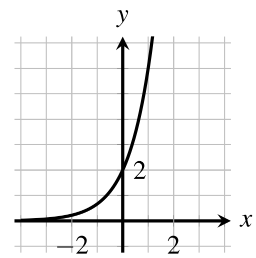
A.



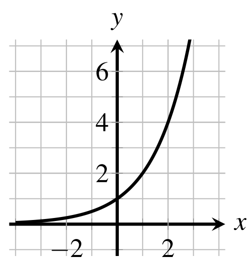
B.



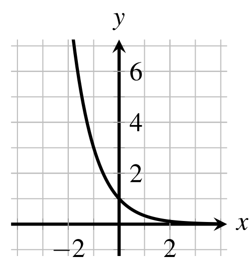
C.



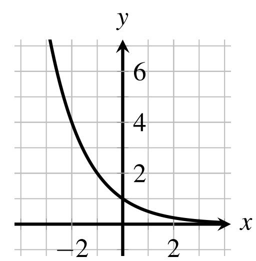
D.



E.



F.



1. $y = 3^x$
2. $y = 2^x$
3. $y = 2(3)^x$
4. $y = 3(2)^x$
5. $y = 2^{-x}$
6. $y = 3^{-x}$

11.6 Chapter Review - No Calculator

After you finish, check your answers on page 34. Full explanations are on page 36.

1

The expression $\sqrt{63} \cdot \sqrt[3]{56}$ is equivalent to which of the following?

A) $5\left(\sqrt[6]{7}\right)^6$

B) $5\left(\sqrt[5]{7}\right)^6$

C) $6\left(\sqrt[5]{7}\right)^6$

D) $6\left(\sqrt[6]{7}\right)^5$

2

The function $M(t) = 3(2)^t$ models the spread of mold after t days. Which of the following models the spread of mold after h hours?

A) $M(h) = 3(2)^h$

B) $M(h) = 3(2)^{24h}$

C) $M(h) = 3(2)^{\frac{h}{24}}$

D) $M(h) = 3(2)^{h+24}$

3

In the complex number system, which of the following is equivalent to $(4+5i)-(3+4i)$? (Note: $i = \sqrt{-1}$)

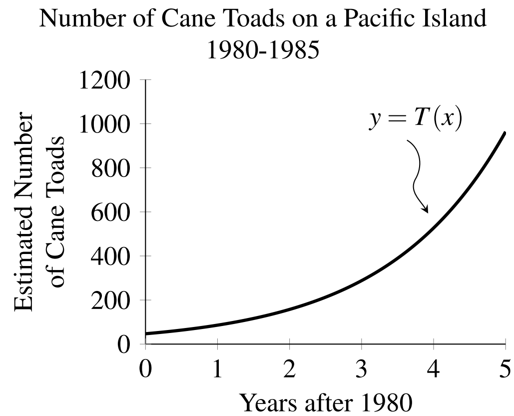
A) 0

B) $1+i$

C) $1+9i$

D) $-7-9i$

4



The graph of the exponential function T above gives the estimated number of cane toads on an isolated island x years after 1980. Which of the following best models the function $T(x)$?

A) $T(x) = 1.83(47)^x$

B) $T(x) = 1.83(47)^{-x}$

C) $T(x) = 47(1.83)^x$

D) $T(x) = 47(1.83)^{-x}$

5

$$\left(2x^2\right)^{\frac{3}{2}}$$

Which of the following is equivalent to the expression above?

A) $x^2 \cdot \sqrt{8}$

B) $2x^3 \cdot \sqrt{2}$

C) $2x^3 \cdot \sqrt{8}$

D) $4x^3 \cdot \sqrt{2}$

6

$$P(t) = 6.21(0.98)^t$$

The function P above models the population of a small town, in thousands, where t is the number of years after 1960 and $0 \leq t \leq 80$. Which of the following is the most likely meaning of 6.21 in this context?

A) The population of the town, in thousands, in 1960.

B) The predicted population of the town, in thousands, t years after 1960.

C) The estimated percent by which the population of the town decreased each year after 1960.

D) The estimated decrease in the population, in thousands, each year after 1960.

7

On the island of Mljet, there were no mongooses before 1910 when 11 small Indian Mongooses were introduced in order to control the number of snakes. The number of small Indian Mongooses then increased by an average of 62.7% per year. If the population grew exponentially at this rate, which of the following functions best models the population $m(t)$ of the small Indian Mongooses on the island t years after 1910 ?

A) $m(t) = (1.627 \times 11)^t$

B) $m(t) = 11(2)^{1.627t}$

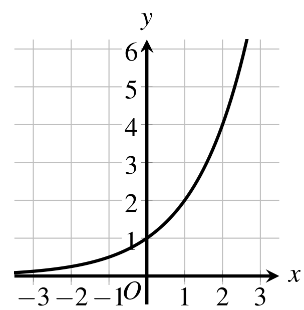
C) $m(t) = 11(1.627)^t$

D) $m(t) = 1.627(11)^t$

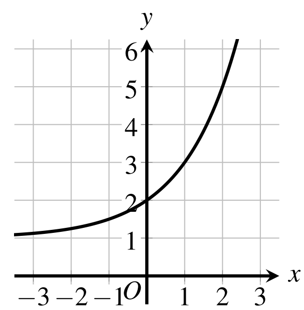
8

Which of the following is the graph of the equation $y = 2^x + 1$ in the xy-plane?

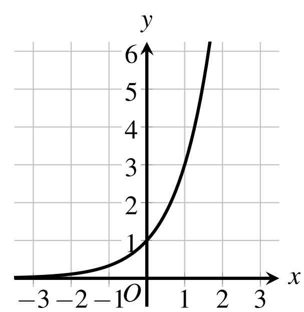
A)



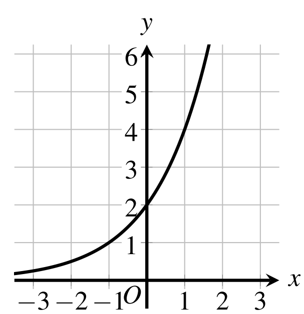
B)



C)



D)



9

$$\frac{a^{-3}b^{\frac{1}{3}}}{a^{\frac{1}{2}}b^{-2}}$$

Which of the following is equivalent to the expression above, where $a > 1$ and $b > 1$?

A) $\dfrac{b\sqrt[3]{b}}{a\sqrt{a}}$

B) $\dfrac{b^2\sqrt[3]{b}}{a^3\sqrt{a}}$

C) $\dfrac{b^4\sqrt[3]{b}}{a^4\sqrt{a}}$

D) $\dfrac{b^3\sqrt{b}}{a^2\sqrt[3]{a}}$

10

$$A(t) = 8(2)^{\frac{t}{3}}$$

The function A above predicts the number of square miles that contain the invasive plant creeping thistle after t years. Based on the function, which of the following statements is true?

A) The predicted square miles that contain the invasive plant doubles three times every year.

B) The predicted square miles that contain the invasive plant doubles every three years.

C) The predicted square miles that contain the invasive plant triples every two years.

D) The predicted square miles that contain the invasive plant triples two times every year.

11

For the positive integer m, which of the following is equivalent to $\sqrt[m]{2}\cdot 5^{\frac{2}{m}}$?

A) $2^{\frac{5}{m}}$

B) $20^{\frac{1}{m}}$

C) $\sqrt[m]{10}$

D) $\sqrt[m]{50}$

12

The half-life of Radium-226 is approximately 1600 years. For a sample of 75 milligrams of Radium-226, which of the following exponential functions best models the amount of Radium-226 $R(t)$, in milligrams, that remains after t years? (The half-life is the length of time needed for half of a radioactive substance to decay, leaving half of the original sample.)

A) $75\left(\frac{1}{2}\right)^{1600t}$

B) $1600\left(\frac{1}{2}\right)^{75t}$

C) $75\left(\frac{1}{2}\right)^{\frac{t}{1600}}$

D) $1600\left(\frac{1}{2}\right)^{\frac{t}{75}}$

13

$$y = 3^x$$

What is the y-intercept of the above equation in the xy-plane?

A) $(0,1)$

B) $(0,3)$

C) $(1,0)$

D) $(1,3)$

14

The population of wolves in a protected habitat is modeled by the function $W(t) = 120(1.03)^{\frac{t}{4}}$ where t is the number of years after 2020. The wolves are predicted to increase by 3% every m months. What is the value of m ?

A) 3

B) 4

C) 12

D) 48

15

$$m^{\frac{n}{3}} = 8$$

In the equation above, m and n are positive integers. What is one possible value of n ?

16

If $(12-5i)(2-3i)$ is written in the form $a+bi$, where a and b are real numbers, what is the value of a ? (Note: $i = \sqrt{-1}$)

17

$$x^{\frac{a}{b}} = \frac{x \cdot \sqrt[3]{x^5}}{\sqrt{x}}$$

In the equation above, $x > 1$. What is the value of $\frac{a}{b}$?

18

Alyssa purchased a painting with a value of \$300. Each year, the value of the painting increased by 20%. If the value of the painting after 2 years is $300n$ dollars, what is the value of n ?

19

$$5i^4 - 7i^2 + 12$$

In the complex number system, what is the value of the expression above?

20

For the function $f(x) = a^x$, where a is a constant greater than 1, if $f(5) = 8 \cdot f(2)$, what is the value of a ?

11.7 Chapter Review - Calculator Allowed

After you finish, check your answers on page 34. Full explanations are on page 41.

1

Amanda started an annual charity bake sale. The first year, she made 40 cookies for the bake sale. Each year for the next 5 years, she made double the number of cookies for the bake sale as she had the previous year. If $C(t)$ is the number of cookies Amanda made for the bake sale t years after the first bake sale, which of the following statements best describes the function C ?

A) The function C is a decreasing linear function.

B) The function C is an increasing linear function.

C) The function C is a decreasing exponential function.

D) The function C is an increasing exponential function.

2

For each increase by 1 in the value of x, the exponential function h decreases by 60%. If $h(0) = 4$, which of the following could be the equation for h ?

A) $h(x) = 0.6(4^x)$

B) $h(x) = 0.4(4^x)$

C) $h(x) = 4(0.6^x)$

D) $h(x) = 4(0.4^x)$

3

$$P(t) = 300t$$
$$P(t) = 100(2^t)$$

A biologist is deciding between two different models above to predict the total population, P, of snakes on an island t years after the first measurement. How many more snakes are predicted by the exponential model than by the linear model 4 years after the first measurement?

A) 200

B) 400

C) 1,200

D) 1,600

4

For the following four types of checking accounts, which option would result in exponential growth of the money in the account?

A) Every year, 3% of the initial deposit is added to the account.

B) Every year, 2% of the initial deposit and \$200 is added to the account.

C) Every year, 1% of the current value is added to the account.

D) Every year, \$500 is added to the account.

5

$$M(t) = 9(1.74)^t$$

A plate of agar is placed in a Petri dish and used to grow an initial sample of 9 square centimeters of mold. The mold grows on the plate according to the function above, where M is the total area covered in mold, in square centimeters, after t hours. Which of the following gives the total amount that the area, in square centimeters, has increased after 15 hours?

A) $M(0)$

B) $M(15)$

C) $M(15) - M(0)$

D) $\dfrac{M(15)}{M(0)}$

6

x	$f(x)$
0	1
1	3
2	9

The graph of the exponential function $f(x)$ in the xy-plane passes through the points shown in the table above. Which of the following can NOT be true of the line $y = mx$ for any value of m ?

A) The line $y = mx$ does not intersect the graph of f.

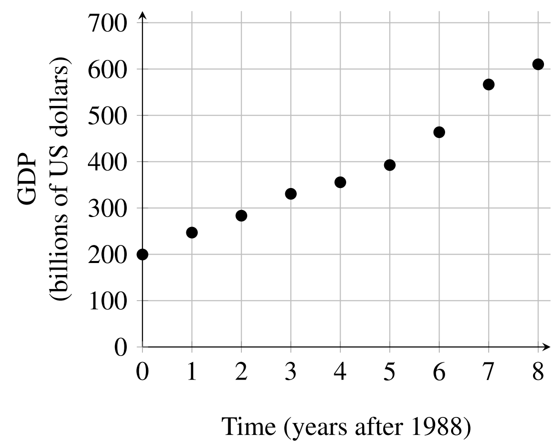
B) The line $y = mx$ intersects the graph of f at exactly one point.

C) The line $y = mx$ intersects the graph of f at exactly two points.

D) The line $y = mx$ intersects the graph of f at exactly three points.

7

The scatterplot below shows the GDP of South Korea, in billions of US dollars, from 1988 to 1996.



Which of the following function best models the GDP of South Korea, G, in billions of US dollars, where t is the number of years after 1988?

A) $G(t) = 0.14^t$

B) $G(t) = 1.14^t$

C) $G(t) = 199(0.14)^t$

D) $G(t) = 199(1.14)^t$

8

Which of the following, for the pair of variables given, describes an exponential relationship?

A) A boat's speed B increases at a constant rate of 3 meters per second for each second, s.

B) As a pool is drained, the amount of water w in the pool decreases by 7 gallons each minute m.

C) Each second s, the temperature T of a piece of molten steel decreases by 30%.

D) For every increase in the number of bookshelves, s, the number of books a bookstore can hold B increases by 120.

9

$$f(x) = ab^x$$
$$g(x) = mx + b$$

The functions above are graphed in the xy-plane, where $y = f(x)$ and $y = g(x)$. a, b, and m are constants, with $b > 1$. Both graphs contain the points (2, 4) and (3, 5). If $f(d) < g(d)$, where $1 < d < 5$, which of the following must be true?

A) $1 < d < 2$

B) $2 < d < 3$

C) $3 < d < 4$

D) $4 < d < 5$

10

The price of a certain plot of land increased by 5% from the previous year for 20 consecutive years. If x is the number of years since the start of the increase in price, which of the following expressions could model the cost of the plot of land over this 20 year time period?

A) $1.5x + 20,500$

B) $1.05x + 20,500$

C) $20,500(1.5)^x$

D) $20,500(1.05)^x$

11

Carl deposited \$500 in a bank account that earns 3% interest compounded annually. He uses the function $A(t) = 500(r)^t$ to calculate the value of the account after t years. What is the value of r in the function?

12

An immunologist is studying the growth of a bacteria in a controlled environment. At the start of each day, there are twice as many bacteria as there were at the start of the previous day. If there are 50 bacteria at the start of the first day, how many bacteria will there be at the start of the seventh day?

13

A geologist is modeling the growth of a desert that has an area of 1,000 square miles using two different models. In the first model, the area of the desert grows by 1% every year. In the second model, the area of the desert grows by 1.5% every year. After 5 years, how much larger, in square miles, is the desert in the second model than in the first model? (Round your answer to the nearest square mile.)

14

Years after 1980	Population
0	40,023
10	17,348
20	7,559
30	3,205
40	1,414

The population of a small US county was checked every ten years in a census. The population decrease, starting in 1980, is modeled by the function $P(t) = 40000r^{\frac{t}{10}}$. If P approximates the population in the table to within 100 people, what is the value of r, rounded to the nearest tenth?

15

If the equation $y = 2^x + 8$ is graphed in the xy-plane, what is the y-coordinate of the y-intercept of the graph?

Chapter 11 Exponents, Exponential Growth, and Imaginary Numbers Answers

12.1 Answers and Explanations

What Should I Work On?

1. D
2. A
3. A
4. D
5. A
6. D
7. D
8. B
9. C
10. B
11. D

Practice Drill 1
Exponent Rules

1. 13
2. 8
3. 40
4. 6
5. 36
6. 103
7. 12
8. 11
9. 8
10. 27

Practice Drill 2
Radicals and
Fractional Exponents

1. C
2. B
3. H
4. F
5. E
6. I
7. B
8. K
9. C
10. J

Practice Drill 3
Imaginary Numbers

1. $11+14i$
2. $19-i$
3. $7+7i$
4. $8+i$
5. 0
6. $-24-16i$
7. $7+22i$
8. $36-3i$
9. 2
10. $6-12i$

Practice Drill 4
Linear and
Exponential Functions

1. A
2. C
3. A
4. D
5. D
6. A
7. B
8. D
9. B
10. C

Practice Drill 5
Exponential Formula

1. $y=30(2)^x$
2. $y=350(0.95)^x$
3. $y=34.9(1.03)^x$
4. $y=12(3)^x$
5. $y=12(0.2)^x$
6. $y=65000(1.06)^x$
7. $y=1664(0.5)^x$
8. $y=23(1.023)^x$
9. $y=4(0.5)^x$
10. $y=760(0.99988)^x$

Practice Drill 6
Time Intervals

1. 3 times per day
2. 2 times per year
3. Once per 5 years
4. 12 times per year
5. Once per 898 years
6. $y=18(2)^{\frac{x}{7}}$
7. $y=40(3)^{4x}$
8. $y=29(1.11)^{\frac{x}{2}}$
9. $y=650,000(0.995)^{3x}$
10. $y=9\left(\frac{1}{2}\right)^{4x}$

Practice Drill 7
Exponential Graphs

1. B
2. D
3. C
4. A
5. F
6. E

Chapter 11 Review
No Calculator

1. D
2. C
3. B
4. C
5. B
6. A
7. C
8. B
9. B
10. B
11. D
12. C
13. A
14. D
15. 1, 3, or 9
16. 9
17. $\frac{13}{6}$ or 2.16 or 2.17
18. 1.44
19. 24
20. 2

Chapter 11 Review
Calculator Allowed

1. D
2. D
3. B
4. C
5. C
6. D
7. D
8. C
9. B
10. D
11. 1.03
12. 3200
13. 26
14. 0.4
15. 9

Chapter 11 Exponents, Exponential Growth, and Imaginary Numbers Explanations

Explanations for Chapter 11 What Should I Work On?

1.	$a^7=2$ $\left(a^7\right)^3=2^3$ $a^{21}=8$	The question asks for a^{21}. Is there a way to go directly from a^7 to a^{21}? Cube both sides. Answer choice (D).

2.

$\left(64x^8y^6\right)^{\frac{1}{3}}$	Rewrite the $\sqrt[3]{\ }$ as an exponent of $\frac{1}{3}$ power.
$64^{\frac{1}{3}}\left(x^8\right)^{\frac{1}{3}}\left(y^6\right)^{\frac{1}{3}}$	Since each part is being multiplied, we can distribute the $\frac{1}{3}$ power.
$4x^{\frac{8}{3}}y^{\frac{6}{3}}$	$64^{\frac{1}{3}} = \left(4^3\right)^{\frac{1}{3}} = 4$ You can also think of the exponents in the original form, as a cube root: $\sqrt[3]{64} = 4$, since $4^3 = 64$. For the x and y terms, when you have an exponent raised to another power, multiply the exponents.
$4x^{\frac{8}{3}}y^2$	Simplify $\frac{6}{3} = 2$, answer choice (A).

3.

$5i(8) - 5i(3i) + 2i$	At first, just treat i like any other variable, and we'll distribute and combine like terms as usual. Distribute the $5i$ to each term in the parenthesis.
$40i - 15i^2 + 2i$	Simplify.
$42i - 15(-1)$	Since $i = \sqrt{-1}$, $i^2 = -1$. Also, combine the like terms $40i$ and $2i$.
$15 + 42i$	$-(-15) = 15$. $42i + 15 = 15 + 42i$, answer choice (A). The SAT will normally put the real part (in this case, 15) before the imaginary part (in this case, $42i$), so you'll often have to rearrange your answer like this into the form $a + bi$.

4.

Increasing exponential	Since the revenue is increasing, eliminate answer choices (A) and (B). The revenue doesn't increase by the same *amount* each year: for instance, it's not always increasing by \$100. Instead, it's always increasing by the same *percent*: the higher our revenue, the more extra revenue we're getting. That's an exponential function, answer choice (D).

5.

$20(3)^t$	Use the standard exponential growth formula $y = ab^x$, where a is the initial amount (20 square feet of algae), b is the rate (3 times per day) and x is the time, t. Answer choice (A).

6.

$b = 1.06$	Using the standard exponential growth formula $y = ab^x$, where a is the initial amount and b is the rate of change, we know that the initial amount is 31.96 grams and 1.06 is the rate of change. Every time x, the length, increases by 1, we'll multiply the weight by 1.06.
$1.06 = 106\% = 100\% + 6\%$	That's an increase of 6% each time. Eliminate (A) and (B). x is the length, so each time our length increases by 1, the weight gets 6% larger, answer choice (D).

7.

$y = 75,000(1.2)^x$	Start with the standard exponential growth formula $y = ab^x$, where a is the initial amount (75,000 people) and b is the growth rate. Since the city is expected to grow by 20%, the population each year will be 100% of the old population plus another 20% of the population: 120% total, and $b = 1.2$. Eliminate answer choices (A) and (B).
$y = 75,000(1.2)^{\frac{t}{10}}$	The planner is estimating that the population will grow 20% every 10 years. Since it's changing once per 10 years, our exponent should be a fraction: $\frac{t}{10}$, answer choice (D). Remember that we can also find when the exponent would be 1: we want the population to be $75,000(1.2)^1$, an increase of 20%, after 10 years. What should our exponent be so that, when we use $t = 10$, the entire exponent is 1? $\frac{t}{10} = \frac{10}{10} = 1$.

8.

$R(1) = 3100(1.08)^1$

Let's plug in a number. If a single year has passed, then $t = 1$ and there should be a total of $3100(1.08)^1$ rabbits on the island. Even if we measure the rabbits every quarter year, the number of rabbits after a year shouldn't change: it should still be $3100(1.08)$. So, 4 quarter years (also known as "1 year") there should be $3100(1.08)$ rabbits. Let's plug in $q = 4$ and see which one gives us that exact same number of rabbits.

A) $3100(1.08)^{4(4)}$ — Nope, eliminate (A).

B) $3100(1.08)^{\frac{4}{4}} = 3,100(1.08)^1$ — That works, keep (B).

C) $3100(1.02)^{4(4)} = 3,100(1.02)^{16}$ — Nope, $1.02^{16} \neq 1.08^1$, eliminate (C).

D) $3100(1.02)^4$ — This is actually pretty close! $1.02^4 = 1.0824$ish, which is close to 1.08...but isn't 1.08. Eliminate (D), and the answer is (B).

9.

$y = ab^x$

Compare this equation to the standard exponential function, $y = ab^x$. a is the initial amount. (They're using h instead of a, but notice that it means the same thing: h doesn't have an exponent on it, so it's our starting amount.) Here, the county started with a population of 130,000, so $h = 130,000$, eliminate (A) and (B).

The population is decreasing, which means that our exponent needs to be negative. (Or, we need a fraction as our rate. Since the function already has $b = 10$, the only way to make the population go down is with a negative exponent, so 10 becomes $\frac{1}{10}$, and the answer is (C). Or, if you're not sure, you can also try out the values from the table with your calculator: try $t = 15$ and see which answer gives you something close to $82,323$.

10.

Exponential increase

Since the number of transistors doubles every 2 years, this is an exponential function: it doesn't increase by the same amount, it increases by more and more each time. So the graph should get larger and larger as time continues, answer choice (B).

11.

(2, 15ish)

Try out a point from the graph. The graph isn't super easy to read, unfortunately, but when $x = 2$ the y value looks like it's somewhere around 15. Maybe a bit higher.

A) $f(2) = 3^{-2} = \frac{1}{9}$ — Trying out $x = 2$ to see if we get $y = 15$ish. And we do not. Way too small. Remember that negative exponents are just fractions: $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

B) $f(2) = 3^2 = 9$ — Closer, but still a little too small: the point on the graph where $x = 2$ is definitely larger than 10, so eliminate (B).

C) $f(2) = 4^{-2} = \frac{1}{4}$ — Ugh, another fraction. No.

D) $f(2) = 4^2 = 16$ — 16 is pretty close to our guess for the y value, and looking at the graph again, yes, (2,16) is what I meant, not (2,15), I clearly said (2,16), that was always my answer.

Explanations for Chapter 11 Review - No Calculator

1.	$\sqrt{9(7)} \cdot \sqrt[3]{8(7)}$	Break apart each root. For the square root, find something that goes into 63 that we can take the square root of: $9 \times 7 = 63$, so rewrite $\sqrt{63} = \sqrt{9(7)}$. We'll do the same thing with the cube root: find a number that goes into 56 that we know the cube root of. 8, 27, and 64 are all easy numbers to take the cube root of, and only 8 goes into 56, so rewrite $\sqrt[3]{56} = \sqrt[3]{8(7)}$.
	$\sqrt{9}\sqrt{7} \cdot \sqrt[3]{8}\sqrt[3]{7}$	Whenever you have multiplication under roots, you can break apart the roots.
	$3\sqrt{7} \cdot 2\sqrt[3]{7}$	Simplify the roots: $\sqrt{9} = 2$ and $\sqrt[3]{8} = 2$.
	$6\sqrt{7}\sqrt[3]{7}$	Combine like terms. Eliminate (A) and (B), since the coefficient in our answer is 6.
	$6 \cdot 7^{\frac{1}{2}} \cdot 7^{\frac{1}{3}}$	Rewrite the roots using exponents.
	$6 \cdot 7^{\frac{1}{2}+\frac{1}{3}}$	When multiplying, add the exponents.
	$6 \cdot 7^{\frac{5}{6}}$	Give the fractions common denominators to add them: $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$.
	$6 \cdot \sqrt[6]{7^5}$	Rewrite the fractional exponent using $\frac{power}{root}$. Answer choice (D).

2.	$M(1) = 3(2)^1 = 6$	Try out $t = 1$, the amount of mold after 1 day. We should get the same amount of mold after 24 hours, when $h = 24$.
	A) $3(2)^{24}$	Nope, that's much, much bigger than $3(2)$. Eliminate (A).
	B) $3(2)^{24(24)}$	This is even larger. This is worse. Much worse. Eliminate (B).
	C) $3(2)^{\frac{24}{24}} = 3(2)^1$	Okay, (C) works.
	D) $3(2)^{24+24} = 3(2)^{48}$	Also too large. Eliminate (D), and the answer is (C).
		OR
	$3(2)^{\frac{h}{24}}$	The original equation gives how much mold spreads after a day: it doubles every day, since $b = 2$. It doubles once *per* day, which is once *per* 24 hours, so divide the exponent by 24.

3.	$(4+5i) - (3+4i) = 4 + 5i - 3 - 4i$	Distribute the minus sign to each term in the second set of parentheses.
	$1 + i$	Combine like terms. Answer choice (B).

4.	$a = 50$ish	Since the function is an increasing exponential, eliminate anything with a negative exponent: answer choices (B) and (D) are gone. Using the form $y = ab^x$, where a is the initial value, compare answer choices (A) and (C). For (A), $a = 1.83$, which seems too small for an initial value: the line seems to cross the axis at somewhere below 100. Maybe 50ish. Eliminate (A), leaving (C). (C) has $a = 47$, which matches the graph.

5.	$\left(2x^2\right)^{\frac{3}{2}} = 2^{\frac{3}{2}} \left(x^2\right)^{\frac{3}{2}}$	Distribute the $\frac{3}{2}$ exponent to each piece.
	$\sqrt{2^3}x^3$	Rewrite $2^{\frac{3}{2}}$ using $\frac{power}{root}$: it's 2 to the 3rd power and the 2nd root, cubed and square rooted. For the x term, when raising something with an exponent to another power, multiply the exponents.
	$\left(x^2\right)^{\frac{3}{2}} = x^{\frac{2}{1} \cdot \frac{3}{2}} = x^3$	Just looking at the x term, we can eliminate answer choice (A).
	$\sqrt{2^2}\sqrt{2}x^3$	Break apart $\sqrt{2^3}$ as two separate square roots, so we can take the square root of as much as possible.
	$2\sqrt{2}x^3$	Simplify $\sqrt{2^2} = 2$. Answer choice (B).

6.	$a = 6.21$	The equation is of the form $y = ab^x$, where a is the initial amount and b is the rate of change. $6.21 = a$, the initial population at $t = 0$. Since the function defines t as "the number of years after 1960," 6.21 represents the population, in thousands, of the town at that starting point, 1960. Answer choice (A).
7.	$m(t) = 11(1.627)^t$	Since the number of mongooses grows by 62.7% each year, this is an exponential function, and we can use the equation $y = ab^x$. Our initial number of small Indian Mongooses is 11, so $a = 11$, and we can eliminate (A) and (D). Our rate of change is 62.7%, so each year there will be 100% of the previous year's mongooses plus 62.7% more mongooses, for a total of 167.2% of the previous year's population, so $b = 1.672$, and the answer is (C).
8.	$y = 2^0 + 1 = 1 + 1 = 2$	Start by finding the y-intercept, where $x = 0$. Plug $x = 0$ in to the equation to get that $y = 2$, and our graph should include the point $(0,2)$: eliminate (A) and (C).
	$y = 2^1 + 1 = 2 + 1 = 3$	Now plug in $x = 1$ to find that $y = 3$, and the point $(1,3)$ should be on the graph: eliminate (D), leaving answer choice (B).
9.	$\frac{a^{-3}b^{\frac{1}{3}}}{a^{\frac{1}{2}}b^{-2}} = a^{-3-\frac{1}{2}}b^{\frac{1}{3}-(-2)}$	When dividing, subtract the exponents.
	$a^{-\frac{7}{2}}b^{\frac{7}{3}}$	Simplify. $-3 - \frac{1}{2} = -\frac{6}{2} - \frac{1}{2} = \frac{7}{2}$ and $\frac{1}{3} - (-2) = \frac{1}{3} + \frac{6}{3} = \frac{7}{3}$
	$\frac{b^{\frac{7}{3}}}{a^{\frac{7}{2}}}$	A negative exponent just means to flip the fraction, so put the a with the negative exponent in the denominator of the fraction.
	$\frac{b^{\frac{6}{3}}b^{\frac{1}{3}}}{a^{\frac{6}{2}}a^{\frac{1}{2}}}$	Split apart your fractions: basically, take out the whole number part of each exponent, since that will simplify in our roots: $\sqrt{a^7} = \sqrt{a^2 \cdot a^2 \cdot a^2 \cdot a} = a \cdot a \cdot a \cdot \sqrt{a} = a^3\sqrt{a}$.
	$\frac{b^2b^{\frac{1}{3}}}{a^3a^{\frac{1}{2}}}$	Answer choice (B).
		OR
	$a^{-3-\frac{1}{2}}b^{\frac{1}{3}-(-2)} = a^{-3\frac{1}{2}}b^{2\frac{1}{3}}$	After subtracting, you can leave your answers as mixed numbers.
	$\frac{b^{2\frac{1}{3}}}{a^{3\frac{1}{2}}}$	A negative exponent just means to flip the fraction, so put the a with the negative exponent in the denominator of the fraction.
	$\frac{b^2b^{\frac{1}{3}}}{a^3a^{\frac{1}{2}}}$	When multiplying, you add the exponents, so you can also break up added exponents as separate multiplied pieces.
	$\frac{b^2\sqrt[3]{b}}{a^3\sqrt{a}}$	Rewrite each fractional exponent as a root.
10.	$b = 2$	Since this is an exponential function of the form $y = ab^x$, we know that $a = 8$ and $b = 2$. That means there was an initial area of 8 square miles that contained creeping thistle, and that amount doubled after some amount of time. Eliminate (C) and (D).
	$A(3) = 8(2)^{\frac{3}{3}}$	Let's think what it would take to make the exponent 1. If $t = 3$, then the amount of creeping thistle would double. So, it takes 3 years for it to double, answer choice (B).
		OR
	$A(t) = 8(2)^{\frac{t}{3}}$	Since the exponent is divided by 3, the area doubles once *per* 3 years. Answer choice (B).

11.	$\sqrt[2]{2}\cdot 5^{\frac{2}{2}}$	Since we have variables in the answer choices, try out an easy number for m. I'll try $m=2$, since that gives us some square roots to deal with.
	$\sqrt{2}\cdot 5 = 5\sqrt{2}$	Simplify. Now we have an answer, $5\sqrt{2}$, so plug in $m=2$ into each answer choice to try to get $5\sqrt{2}$.
	A) $2^{\frac{5}{2}} = \sqrt{2^5} = 4\sqrt{5}$	Nope, this isn't the same as $5\sqrt{2}$, so eliminate (A).
	B) $20^{\frac{1}{2}} = \sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$	This doesn't work either. Eliminate (B).
	C) $\sqrt[2]{10} = \sqrt{10}$	Eliminate (C).
	D) $\sqrt[2]{50} = \sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$	And the answer is (D).
		OR
	$2^{\frac{1}{m}}\cdot 5^{\frac{2}{m}}$	Rewrite the root as a fractional exponent.
	$2^{\frac{1}{m}}\cdot 5^{\frac{2}{m}}$	We have different bases, so we can't just combine our exponents. We can separate out the $5^{\frac{2}{m}}$ as $5^{\frac{1}{m}+\frac{1}{m}} = 5^{\frac{1}{m}}5^{\frac{1}{m}}$
	$2^{\frac{1}{m}}5^{\frac{1}{m}}5^{\frac{1}{m}} = (2\cdot 5\cdot 5)^{\frac{1}{m}}$	Now that everything has the same exponent, you can combine them all under the same exponent.
	$(50)^{\frac{1}{m}} = \sqrt[m]{50}$	Simplify and rewrite the fractional exponent as a root. Answer choice (D).
12.	$y = 75\left(\frac{1}{2}\right)^x$	Fill out what you know into $y=ab^x$. The initial amount is 75 milligrams, so $a=75$. The sample is decreasing by $\frac{1}{2}$ each time, so $b=\frac{1}{2}$. Eliminate (B) and (D).
	$75\left(\frac{1}{2}\right)^{\frac{1600}{1600}}$	When $t=1600$, the Radium-226 will decay by one half exactly one time, so we want an answer that gives us an exponent of 1 when $t=1600$, and the answer is (C). Or, you can remember that if something happens once *per* 1600 years, the exponent needs to be *divided* by 1600.
13.	$y = 3^0 = 1$	To find the y-intercept, plug in $x=0$. Eliminate (C) and (D). Since anything raised to the 0th power is 1, the y-intercept is at (0, 1), answer choice (A).
14.	$t=4$	Since the equation is in terms of years, start by finding how many years it takes for the wolf population to increase by 3%. The exponent is $\frac{t}{4}$, which means that it increases by 3% once *per* 4 years. (Or, when $t=4$ the exponent will be 1 and the population will be $120(1.03)$, a 3% increase. So it takes 4 years to grow by 3%.)
	$4(12) = 48$	Now convert from years to months: there are 12 months in a year, so if $t=4$ then $m=48$, answer choice (D).

15.	$8^{\frac{3}{3}} = 8$	Try out some values. Since we want to make the left side equal 8, try out $m = 8$. In that case, we need an exponent of 1, so that $8^1 = 8$. That means that $\frac{n}{3} = 1$ and $n = 3$.
		OR
	$2^{\frac{9}{3}} = 8$	We could also try out $m = 2$, since $2^3 = 8$. (With exponent questions, if you see an 8 it's often best to think of it as 2^3. In that case, we need $\frac{n}{3} = 3$, and $n = 9$.
		OR
	$512^{\frac{1}{3}} = 8$	I guess we could also try out something to that could use the cube root. $\sqrt[3]{512} = 8$, so we could make $m = 512$. We really don't need to, but it works. It's one of the answers, it's just not the number you'd want to try out first, since it's big and clunky. But I'm mentioning it here because it does work: $512^{\frac{1}{3}} = \left(8^3\right)^{\frac{1}{3}} = 8^1 = 8$. So n could equal 1 as well.
	$n = 1, 3$, or 9	The three possible answers for n are therefore $n = 3$, $n = 9$, or $n = 1$.

16.	$(12-5i)(2-3i) = 24-36i-10i+15i^2$	Expand using FOIL.
	$24-46i-15$	Combine like terms. Since $i = \sqrt{-1}$, $i^2 = -1$, and $15i^2 = -15$.
	$9-46i$	Rwrite in the form $a+bi$, and $a = 9$.

17.	$\frac{x\sqrt[3]{x^5}}{\sqrt{x}} = \frac{x \cdot x^{\frac{5}{3}}}{x^{\frac{1}{2}}}$	Rewrite the roots as fractional exponents.
	$\frac{x^{\frac{8}{3}}}{x^{\frac{1}{2}}}$	When multiplying, add the exponents. $x^1 \cdot x^{\frac{5}{3}} = x^{1+\frac{5}{3}} = x^{\frac{3}{3}+\frac{5}{3}} = x^{\frac{8}{3}}$.
	$x^{\frac{8}{3}-\frac{1}{2}} = x^{\frac{13}{6}}$	When dividing, subtract the exponents. $\frac{8}{3} - \frac{1}{2} = \frac{16}{6} - \frac{3}{6} = \frac{13}{6}$.

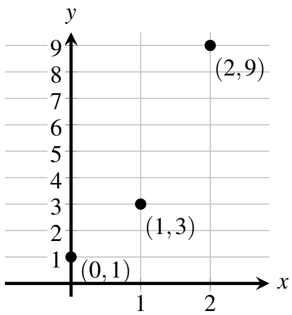
18.	$300(1.2)^2 = 432$	Set up the equation using $y = ab^x$, where the initial amount is $a = 300$ and $b = 1.2$ for a 20% increase per year. You can also just multiply 300 by 1.2 twice: $300(1.2)(1.2) = 432$
	$300n = 432$	The question says the value after 2 years is $300n$ dollars, so set $300n$ equal to 432.
	$n = 1.44$	Divide both sides by 300.
		OR
	$300n = 300(1.2)^2$	Rather than multiplying everything out, set $300n$ equal to the final price, which is 300 plus 20%, plus another 20%, or $300(1.2)^2$.
	$n = 1.2^2 = 1.44$	Divide both sides by 300.

19.	$5i^4 - 7i^2 + 12 = 5(-1)(-1) - 7(-1) + 12$	Since $i = \sqrt{-1}$, $i^2 = -1$, and we can replace each i^2 with -1. Remember that $i^4 = (i^2)(i^2) = (-1)(-1) = 1$.
	$5+7+12 = 24$	Simplify.

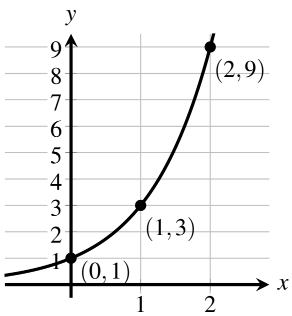
20.	$a^5 = 8 \cdot a^2$	Since the problem says that $f(x) = a^x$, we can use $f(5) = 8 \cdot f(2)$. $f(5) = a^5$ and $f(2) = a^2$.
	$a^3 = 8$	Divide both sides by a^2. $\frac{a^5}{a^2} = a^{5-2} = a^3$.
	$a = 2$	Take the cube root of both sides. You can also rewrite 8 as 2^3, which gives you $a^3 = 2^3$ and $a = 2$.

Explanations for Chapter 11 Review - Calculator Allowed

1.	Increasing exponential	The number of cookies Amanda makes increases: 80 in the second year, 160 in the third year, 320 in the fourth year, and 640 for the fifth year, this is an increasing exponential function, answer choice (D).
2.	$100\% - 60\% = 40\% = 0.4$	Use the function $y = ab^x$. The rate, b is 0.4, since the value of $h(x)$ decreases by 60% each time. Eliminate (A), (B) and (C). The answer is (D). $a = 4$ because the initial value, our y-intercept, is $h(0) = 4$.
3.	$P(4) = 300(4) = 1200$	Start by finding the number of snakes predicted by the linear model by plugging in $t = 4$.
	$P(4) = 100(2^4) = 100(16) = 1600$	Find the number of snakes predicted by the exponential model after 4 years.
	$1600 - 1200 = 400$	Find the difference in the two predictions, answer choice (B).
4.		For exponential growth, the amount of money added each year has to increase. Let's say we started with an initial deposit of \$100 in the account.
	A) $100 + 0.03(100) = 103$	For answer choice (A), after one year we'd gain \$3 to our account.
	$103 + 0.03(100) = 106$	After another year, we'd still only gain \$3 to our account, because we're getting 3% of the initial deposit, not of the current value. Eliminate (A).
	B) $100 + 0.02(100) + 200 = 302$	For answer choice (B), after one year we gained \$202 to our account. Way better than (A), but let's see what happens after another year.
	$302 + 0.02(100) + 200 = 504$	After another year, we'd just gain another \$202 to our account. As with (A), we're earning 2% from our initial deposit, so the amount we earn never changes, never increases. Eliminate (B).
	C) $100 + 0.01(100) = 101$	For answer choice (C), we're earning a whopping \$1 in interest after a year. Not as good as (B), money-wise, but let's see after another year.
	$101 + 0.01(101) = 101 + 1.01 = 102.01$	After another year, we're earning \$1.01 interest. Just a little bit more than we earned the year before. The amount of interest is increasing, so this is an exponential function. (The function would be $y = 100(1.01)^x$, in case you're curious.) The answer is (C).
	D) $100 + 500$	For (D), this is definitely a linear function: every single year we add \$500 into our account. Never more, never less, so this can't be the answer.
5.	(Area after 15 hours) − (initial area)	The question asks for the total amount that the area has increased after 15 hours. The total increase is the area at 15 hours minus whatever the area was at the beginning.
	$M(15) - M(0)$	The area after 15 hours is our function with 15 put in for t: $M(15)$. The initial area is when $t = 0$, so it's an area of $M(0)$, whatever that ends up being. The answer is (C). Notice that we didn't need to actually calculate the area: we just needed to find the function definition of the increase in area.

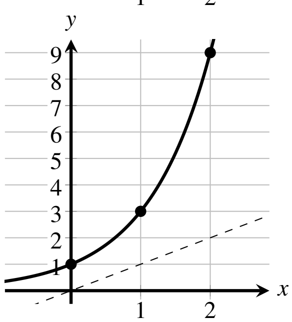


Start by plotting the points, using $f(x) = y$ to get an idea of what the function will look like.

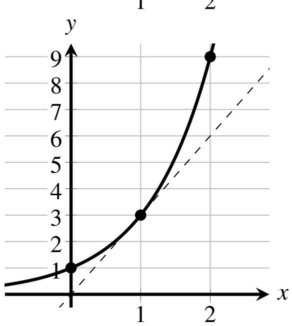


Draw your best guess for the exponential function that passes through the points. This drawing is, obviously, way more accurate than I could actually do by hand, but the only important thing for your drawing is that it goes through those 3 points and it curves a little bit.

6.

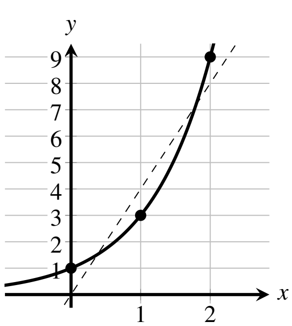


Try out answer choice (A). $y = mx$ is just a straight line that goes through the origin, since the y-intercept (b) is 0. So, is there a straight line we can draw that will never intersect with our graph of f? Sure, if our slope is 1, as shown. Even a slope of 0 would never intersect the graph of f. A bunch of other slopes would also work to avoid intersecting the graph as well, but we just need to find one possibility, and we know that statement (A) is true. Eliminate (A) because the question asks for which one is NOT true.



Okay, then what about (B)? Is it possible to draw a line that only intersects the graph of f exactly once? Sure, if we just barely skim it on the side. Here I've graphed the line $y = 3x$, so they both cross right at $x = 1$. The real answer is just a tad smaller than that: $m \approx 2.9863378...$ will cross it exactly once. it's not super important what the exact number is, or how to find it, the important part is we can definitely draw a line, somewhere, that just crosses it exactly once. Eliminate (B).

6.



Can we draw a line that intersects the exponential graph twice? Definitely, since it's a curved graph. I've drawn $y = 4x$, but there are other possible solutions. Eliminate (C).

That leaves (D). Since there's no way a straight line can cross an exponential 3 times, that's our answer.

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7. $y = 200(1.\text{something})^t$

Use the standard from of an exponential function, $y = ab^x$. The initial value is around 200, so $a = 200$ (or so). Eliminate (A) and (B). Our value for b is harder to tell from the graph: the line goes up, but not by much. But any increase means that b is larger than 1, since we end up with more than 100% of our previous GDP each year. Eliminate (C), which would be a decreasing exponential graph, and the answer is (D).

---

8. For an exponential relationship, the value has to increase by an increasing amount, or decrease by a decreasing amount. For (A), the boat's speed increases by a constant amount: that's a linear function, eliminate (A). Same with (B), since the amount of water changes by the same amount each minute. (D) is also a linear function: the number of books changes by 120 for every new bookshelf. That leaves (C), where the temperature changes by 30% every single time, so each second the temperature is 70% of what it was before.

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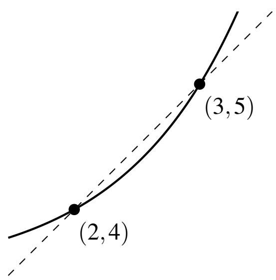
9. 

Draw a rough sketch of the two functions. The solid line is $f(x)$, the exponential function, and the dashed line is $g(x)$, the linear function. I don't know exactly what the functions will look like, but I know they both go through the points (2,4) and (3,5). The linear function will go directly between those two points, but the exponential function will curve a bit, going under the line and then getting bigger and bigger.

$2 < d < 3$

The question says that $f(d) < g(d)$. That's only true in between the two points, where the exponential function curves down below the straight line $g(x)$. If $d < 2$ then $f(d) > g(d)$, and if $d > 3$ then $f(d) > g(d)$. The answer is (B).

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10. $y = a(1.05)^x$

Use the exponential function $y = ab^x$, where a is the initial value and b is the rate of change. Eliminate (A) and (B), both of which are linear functions. In this case, we don't know a, but we know that the value of the land grows by 5% every year, so $b = 105\% = 1.05$, answer choice (D).

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11. $100\% = 3\% = 103\% = 1.03$

Since Carl's bank account grows by 3% each year, each year he has 100% of the amount he had, plus another 3%, for a total of 103%. Convert the percentage into a decimal to get $r = 1.03$.

---

12. $y = 50(2)^x$

Start by writing an exponential equation of the form $y = ab^x$, where a is the initial amount, so $a = 50$, and b is the rate it changes by each day, and $b = 2$.

$y = 50(2)^6 = 50(64) = 3,200$

On the start of the seventh day, a total of 6 days will have passed since the first day, and $x = 6$.

OR

Day 1	50
Day 2	100
Day 3	200
Day 4	400
Day 5	800
Day 6	1600
Day 7	3200

Since there's only seven days to keep track of, just list out the number of bacteria for each day. It doesn't take too long, and you're much less likely to make a mistake.

13.

$y = 1000(1.01)^5 = 1,051.01... = 1,051$	Find the area of the desert using the first model with the equation $y = ab^x$, where $a = 1000$, the initial area of the desert, and $b = 1.01$, a growth of 1%. After 5 years, $x = 5$. Round your answer to the nearest mile.
$y = 1000(1.015)^5 = 1077.28\ldots = 1,077$	Now use $b = 1.015$ to find the growth of the desert in the 1.5% model and round your answer to the nearest mile.
$1077 - 1051 = 26$	Find the difference in the two models.

14.

$17,300 = 40000(r)^{\frac{10}{10}}$	Use the data given in the table in the equation. We already know the initial value is around 40,000, both because it's the population at 1980 and because it's our a term in the $y = ab^x$ equation given. Use the next point, 10 years after 1980, when the population was 14,348. Since the question says the function is accurate to 100 people, you can round 14,348 to 14,300 if that's easier.
$17,300 = 40000r$	Simplify: $(r)^{\frac{10}{10}} = r^1 = r$.
$0.4325 = r$	Divide both sides by 40000 to isolate r.
$r = 0.4$	Round to the nearest tenth.

15.

$y = 2^0 + 8$	To find the y-intercept, plug in $x = 0$.
$y = 1 + 8 = 9$	Simplify, and the y-intercept is at $(0, 9)$.

12.2 Drill Explanations

Explanations for Chapter 11 Practice Drill 1 - Exponent Rules

1.	$(x^5)(x^8) = x^{5+8} = x^{13}$	When multiplying, add the exponents. $a = 13$.
2.	$\frac{x^9}{x} = \frac{x^9}{x^1}$ $\frac{x^9}{x^1} = x^{9-1} = x^8$	Anything without an exponent has a power of 1. When dividing, subtract the exponents. $a = 8$.
3.	$\left(x^5\right)^8 = x^{5(8)} = x^{40}$	When raising an exponent to another power, multiply the exponents. $a = 40$.
4.	$\frac{(x^7)(x^3)}{x^4} = \frac{x^{10}}{x^4}$ $x^{10-4} = x^6$	Start with the top of the fraction. When multiplying, add the exponents, so $(x^7)(x^3) = x^{10}$. When dividing, subtract the exponents. $a = 6$.
5.	$\left(\frac{(x^6)(x^{10})}{x^4}\right)^3 = \left(\frac{x^{16}}{x^4}\right)^3$ $\left(x^{16-4}\right)^3 = \left(x^{12}\right)^3$ $x^{12(3)} = x^{36}$	Start wit the inside top o the fraction. When multiplying, add the exponents, so $(x^6)(x^{10}) = x^{6+10} = x^{16}$. When dividing, subtract the exponents. When raising an exponent to another power, multiply the exponents. $a = 36$.
6.	$x^a(x^5) = \left(x^9\right)^{12}$ $x^{a+5} = \left(x^9\right)^{12}$ $x^{a+5} = x^{(9)12}$ $x^{a+5} = x^{108}$ $a + 5 = 108$ $a = 103$	Start with the left side. When multiplying, add the exponents. On the right side, when raising an exponent to another power, multiply the exponents. Simplify. Set the exponents on each side of the equation equal to each other. Subtract 5 from both sides.
7.	$\frac{x^8}{x^{-4}} = x^{8-(-4)}$ x^{12}	When dividing, subtract the exponents. $8-(-4) = 8+4 = 12$, and $a = 12$.
8.	$(x^{-4})(x^3)^5 = (x^{-4})(x^{3(5)})$ $(x^{-4})x^{15} = x^{-4+15} = x^{11}$	Start with the term on the right. When raising an exponent to another power, multiply the exponents. When multiplying, add the exponents. $a = -19$.

9.	$\left(\frac{x^{-2}}{x^6}\right)\left(x^2\right)^8 = \left(\frac{x^{-2}}{x^6}\right)x^{2(8)}$	Start with the term on the right. When raising an exponent to another power, multiply the exponents.
	$\left(x^{-2-6}\right)x^{16}$	When dividing, subtract the exponents.
	$\left(x^{-8}\right)x^{16} = x^{-8+16} = x^8$	When multiplying, add the exponents. $a = 8$.
10.	$x^5 = 3$ and $a = x^{15}$	Let's see how we can use that x^5. Rewrite $a = x^{15}$ using the fact that 15 = 5(3).
	$a = x^{5(3)} = \left(x^5\right)^3$	When raising an exponent to a power, multiply the exponents. We can also go the other way: if the exponents are multiplied, that's the same as raising our exponent to another power.
	$a = (3)^3 = 27$	Plug in $x^5 = 3$ and simplify.
		OR
	$x = 3^{\frac{1}{5}}$	If you don't see a way to use $x^5 = 3$ and get directly to x^{15}, solve for x. Take the 5th root of both sides. Or, put another way, raise both sides by the $\frac{1}{5}$th power: $\left(x^5\right)^{\frac{1}{5}} = 3^{\frac{1}{5}}$, so $x^1 = 3^{\frac{1}{5}}$.
	$a = x^{15} = \left(3^{\frac{1}{5}}\right)^{15}$	Use $x = 3^{\frac{1}{5}}$ in the equation given.
	$a = \left(3^{\frac{1}{5}(15)}\right) = 3^{\frac{15}{5}} = 3^3 = 27$	When raising an exponent to another power, multiply the exponents.

Explanations for Chapter 11 Practice Drill 2 - Radicals and Fractional Exponents

1.	$x^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}}$	A negative exponent is a fraction: put the entire thing in the denominator.
	$\frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$	A $\frac{1}{2}$ exponent is a square root: the 1st power and the 2nd root. Answer choice (C).
2.	$x^{\frac{2}{3}} = \sqrt[3]{x^2}$	A fractional exponent is $\frac{power}{root}$, so the 2nd power and the 3rd root. Answer choice (B). We could also have written this as $(\sqrt[3]{x})^2$; it doesn't matter whether we do the power on the inside with the root outside, or vice versa. We just happened to have an answer with the power inside, so I matched with that one.
3.	$x^{\frac{4}{3}} = x^{\frac{3}{3}+\frac{1}{3}}$	Split apart $\frac{4}{3}$.
	$x^{\frac{3}{3}}x^{\frac{1}{3}}$	When multiplying, we add the exponents, which means that we can also the opposite: when exponents are added, we can rewrite the expression as multiplication.
	$x\sqrt[3]{x}$	$x^{\frac{3}{3}} = x^1$. We can also rewrite the $x^{\frac{1}{3}}$ as $\sqrt[3]{x}$.
		OR
	$x^{\frac{4}{3}} = \sqrt[3]{x^4}$	Rewrite the $\frac{1}{3}$ fraction as a cubed root.
	$\sqrt[3]{x^3 \cdot x} = x\sqrt[3]{x}$	Since each term is multiplied, split apart the cube root and simplify. Answer choice (H).

4.	$\sqrt{x^4(x^3)} = \sqrt{x^7}$	Simplify the inside of the square root. When you multiply, add the exponents: $4+3=7$.
	$\left(x^7\right)^{\frac{1}{2}}$	Rewrite the square root as an exponent.
	$x^{\frac{7}{2}}$	When you raise an exponent to another power, multiply the exponents: $7 \cdot \frac{1}{2} = \frac{7}{2}$.
	$x^{\frac{7}{2}} = x^{3+\frac{1}{2}}$	Simplify the exponent by breaking apart the fraction into a mixed number: $\frac{7}{2} = 3\frac{1}{2}$.
	$x^{3+\frac{1}{2}} = x^3 x^{\frac{1}{2}}$	When multiplying, add the exponents. Here, we're using that rule in reverse: since the exponents 3 and $\frac{1}{2}$ are added, we can rewrite $x^{3+\frac{1}{2}}$ as multiplication.
	$x^3\sqrt{x}$	Rewrite the remaining $x^{\frac{1}{2}}$ as $\sqrt{x}$, leaving $x^3\sqrt{x}$, answer choice (F).
		OR
	$x^{\frac{7}{2}} = \sqrt{x^7}$	If those last couple of steps were confusing, there's another way to work through it: rewrite the $\frac{1}{2}$ exponent as a square root.
	$\sqrt{x^7} = \sqrt{x^{2+2+2+1}}$	Break apart x^7 using as many x^2 terms as we can: since 2 + 2 + 2 + 1 = 7, we can break x^7 up into $x^{2+2+2+1}$. Those twos will be nice when we have to take the square root in a bit.
	$\sqrt{x^{2+2+2+1}} = \sqrt{x^2 \cdot x^2 \cdot x^2 \cdot x}$	Rewrite the addition in the exponents as multiplication.
	$\sqrt{x^2} \cdot \sqrt{x^2} \cdot \sqrt{x^2} \cdot \sqrt{x}$	We can always break apart multiplication under a square root as separate square roots.
	$x \cdot x \cdot x \cdot \sqrt{x}$	Square roots and squares cancel each other out: $\sqrt{x^2} = x$.
	$x^3\sqrt{x}$	Combine the three x terms to x^3, answer choice (F). If this seems like more steps than you're used to, that's fine: you can go directly from $\sqrt{x^7} = \sqrt{x^6}\sqrt{x} = x^3\sqrt{x}$ once you have the pieces down. I've written out each step here because some of them can be a bit confusing, since it's the opposite way we've been using the exponent rules before now.

5.	$x^{\frac{1}{5}}x^{\frac{1}{2}}$	Rewrite $\sqrt{x}$ as an exponent: $\sqrt{x} = x^{\frac{1}{2}}$
	$x^{\frac{1}{5}+\frac{1}{2}}$	When multiplying, add the exponents.
	$x^{\frac{7}{10}}$	Give the fractions a common denominator to add them: $\frac{1}{5} + \frac{1}{2} = \frac{2}{10} + \frac{5}{10} = \frac{7}{10}$. Answer choice (E).

6.	$x^{\frac{1}{3}}x^{\frac{1}{2}}$	Rewrite the cube root as a $\frac{1}{3}$ power and the square root as a $\frac{1}{2}$ power.
	$x^{\frac{1}{3}+\frac{1}{2}}$	When multiplying, add the exponents.
	$x^{\frac{5}{6}}$	Give the fractions a common denominator to add them: $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$
	$x^{\frac{5}{6}} = \sqrt[6]{x^5}$	Rewrite the fractional exponent using $\frac{power}{root}$. Answer choice (I).

7.	$\frac{\sqrt[3]{x^5}}{x} = \frac{x^{\frac{5}{3}}}{x}$	Rewrite the exponent on the top as a fraction.
	$x^{\frac{5}{3}-1}$	When dividing, subtract the exponents.
	$x^{\frac{2}{3}}$	Subtract $\frac{5}{3} - 1 = \frac{5}{3} - \frac{3}{3} = \frac{2}{3}$.
	$x^{\frac{2}{3}} = \sqrt[3]{x^2}$	Rewrite the fractional exponent using $\frac{power}{root}$: the 2nd power and the 3rd root. Answer choice (B).

8.	$(16x^8)^{\frac{1}{2}} = 16^{\frac{1}{2}}(x^8)^{\frac{1}{2}}$	Distribute the $\frac{1}{2}$ power to each part.
	$4x^4$	Start with $16^{\frac{1}{2}} = \sqrt{16} = 4$. For $(x^8)^{\frac{1}{2}}$, multiply the exponents: $x^{\frac{8}{1}\cdot\frac{1}{2}} = x^4$. Answer choice (K).
9.	$\frac{x^{\frac{5}{2}}}{\sqrt[3]{x^9}} = \frac{x^{\frac{5}{2}}}{x^{\frac{9}{3}}}$	Rewrite the root in the denominator as an exponent: $\sqrt[3]{x^6} = x^{\frac{6}{3}}$.
	$\frac{x^{\frac{5}{2}}}{x^3}$	Simplify the exponent in the denominator: $\frac{9}{3} = 3$.
	$x^{\frac{5}{2}-3}$	When dividing, subtract the exponents.
	$x^{-\frac{1}{2}}$	Simplify: $\frac{5}{2} - 3 = \frac{5}{2} - \frac{6}{2} = -\frac{1}{2}$.
	$x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$	A negative exponent is a fraction, and a $\frac{1}{2}$ exponent is a square root. Answer choice (C).
10.	$\sqrt[3]{4x^3}\,(\sqrt[3]{2x})\left(2\sqrt[3]{x^2}\right) = 2\sqrt[3]{4x^3 \cdot 2x \cdot x^2}$	Since everything (except for the 2 near the end) is cube rooted, we can combine all of the cube roots. Bring the 2 out front and put everything else under a single cube root.
	$2\sqrt[3]{8x^6}$	Combine terms. $4(2) = 8$ and $x^3 \cdot x \cdot x^2 = x^{3+1+2} = x^6$.
	$\sqrt[3]{8}\sqrt[3]{x^6}$	Break apart the cube root again now that everything underneath has been combined.
	$2(2x^2)$	The cube root of 8 is 2, since $2^3 = 8$, and $\sqrt[3]{x^6} = x^{\frac{6}{3}} = x^2$
	$4x^2$	Simplify. Answer choice (J).

Explanations for Chapter 11 Practice Drill 3 - Imaginary Numbers

1.	$(3+12i)+(8+2i) = 11+14i$	Combine like terms.
2.	$(7-8i)+(12+7i) = 19-i$	Combine like terms: $7+12 = 19$ and $-8i+7i = 1i = i$.
3.	$(12+4i)-(5-3i) = 12+4i-5+3i$	Distribute the negative to each term in the second parenthesis. $-(-3i) = 3i$.
	$7+7i$	Combine like terms.
4.	$(4+i)+(5+i^2) = 9+i+i^2$	Combine like terms.
	$9+i-1 = 8+i$	Since $i = \sqrt{-1}$, $i^2 = \left(\sqrt{-1}\right)^2 = -1$. Simplify.
5.	$(-i)^2 - i^2 = i^2 - i^2$	Start with the first $(-i)^2$. Since any negative number times itself is positive, $(-i)^2 = i^2$. Or, put another way, $(-i)^2 = ((-1)(i))^2 = (1)(i^2) = i^2$.
	$i^2 - i^2 = 0$	Any number minus itself is zero. You could also expand out each i^2, so $i^2 - i^2 = -1-(-1) = -1+1 = 0$.

6.	$8i(3i-2)=24i^2-16i$	Distribute the $8i$.
	$-24-16i$	Since $i=\sqrt{-1}$, $i^2=\left(\sqrt{-1}\right)^2=-1$, and $24i^2=-24$.
7.	$(3+2i)(5+4i)=15+12i+10i+8i^2$	Since we're multiplying two binomials, expand with FOIL: First, Outer, Inner, Last.
	$15+22i+8i^2$	Combine like terms.
	$15+22i-8$	Since $i=\sqrt{-1}$, $i^2=\left(\sqrt{-1}\right)^2=-1$, and $8i^2=-8$.
	$7+22i$	Simplify.
8.	$(2-5i)(3+6i)=6+12i-15i-30i^2$	Since we're multiplying two binomials, expand with FOIL: First, Outer, Inner, Last. Don't forget to distribute the negative when multiplying $-5i$!
	$6-3i-30i^2$	Combine like terms.
	$6-3i-(-30)$	Since $i=\sqrt{-1}$, $i^2=\left(\sqrt{-1}\right)^2=-1$.
	$36-3i$	Since $-(-30)=30$, combine like terms.
9.	$(1-i)(1+i)=1+i-i-i^2=1-i^2$	Expand using FOIL. You may have also recognized this as the difference of squares, which gets you right to $(1-i)(1+i)=1^2-i^2=1-i^2$.
	$1-(-1)=1+1=2$	Since $i=\sqrt{-1}$, $i^2=\left(\sqrt{-1}\right)^2=-1$. Simplify.
10.	$12i^3+6=-12i+6=6-12i$	Break apart $i^3=i^2(i)=-1(i)=-1$.

Explanations for Chapter 11 Practice Drill 4 - Linear and Exponential Functions

1.	A) Increasing linear	Since the same number of books are added each month, this is a linear function. The slope is 20 books per month, going up by 20 every single month.
2.	C) Increasing exponential	The number of visitors each month doubles, which means it doesn't increase by the same amount each time, the amount it increases by keeps getting bigger and bigger. We are multiplying the number of visitors by 2 every single month, an exponential function.
3.	A) Increasing linear	Since the same number of people join each month, this is a linear function. The slope is 12 members per year, the same number of people joining each year, a nice straight line.
4.	D) Decreasing exponential	The population decreases by a percentage each year, rather than by the same amount each year. When there's a smaller population, fewer people will leave. It will always be 9% smaller each year, but 9% of a smaller number each time. The graph of population would curve down as it continues to the right, rather than being a straight line.

5.	D) Decreasing exponential	The computer's price goes down by a percentage, rather than by the same amount each year. Every year, as the price is lower, it decreases by the same percentage, which is a percentage of a smaller number, and therefore a smaller dollar decrease in price. The graph would be a curved line.
6.	A) Increasing linear	The price goes up by the exact same amount, $10, each year. The graph of the price would be a straight line.
7.	B) Decreasing linear	The speed of the truck is decreasing by the same amount each second: 3 mph. The graph of the speed would be a straight line with a slope of -3.
8.	D) Decreasing exponential	The size of the wetland isn't decreasing by the same amount; it is decreasing by the same fraction. Every decade there is 50% as much wetland as there was the decade before.
9.	B) Decreasing linear	The value of the company goes down by the exact same amount every single decade. When another 10 years pass? It'll go down by 1.2 million dollars again. The graph of the value of the company would be a straight line.
10.	C) Increasing exponential	The number of voles goes up by more and more each decade. If there were 100 voles in 2000, there were 300 voles in 2010. In 2020, there were 900. Every 10 years, the number triples. The graph would look like a curved line, curving up very quickly.

Explanations for Chapter 11 Practice Drill 5 - Exponential Functions Formula

1.	$y = 30(2)^x$	Using the formula $y = ab^x$, the initial amount a is 30. The number of bacteria double each hour, so we're multiplying by 2, and $b = 2$.
2.	$y = 350(0.95)^x$	Using the formula $y = ab^x$, the initial amount a is \$350. The cost decreases by 5%, which means the price each year is $100\% - 5\% = 95\%$ of the old price, so $b = 0.95$.
3.	$y = 34.9(1.03)^x$	Using the formula $y = ab^x$, the initial amount a is 34.9 million. Since the question says that y is in millions, we can leave out that million part and $a = 34.9$. The company's value increases by 3% every year, so each year the value of the company will be 103% of what it was the year before, and $b = 1.03$.
4.	$y = 12(3)^x$	Using the formula $y = ab^x$, the initial amount a is 12 fish. The number of fish tripled every year, so $b = 3$.

5.	$y = 12(0.2)^x$	Using the formula $y = ab^x$, the initial amount a is 12 ounces of liquid meal (yum). The meal is digested at a rate of 80% per hour, so after a single hour only 20% of the meal will be left, and $b = 0.2$.
6.	$y = 65000(1.06)^x$	Using the formula $y = ab^x$, the initial amount a is \$65,000 per year. The rate of increase is 6% per year, so each year the employee's salary will be 106% of what it was the previous year, and $b = 1.06$.
7.	$y = 1664(0.5)^x$	Using the formula $y = ab^x$, the initial amount a is 1664 boxes. Since the number of boxes goes down by $\frac{1}{2}$ each day, $b = \frac{1}{2} = 0.5$.
8.	$y = 23(1.023)^x$	Using the formula $y = ab^x$, the initial amount a is \$23 per share. That price has increased each year since 2010 by 2.3%, which means that in 2011 the price was 102.3% of what it was in 2010, and every year it's $100\% + 2.3\% = 102.3\%$ of the previous year's price, and $b = 1.023$.
9.	$y = 4(0.5)^x$	Using the formula $y = ab^x$, the initial amount a is 4 milligrams of medication. The amount after a week is $\frac{1}{2}$ what it was, and $b = \frac{1}{2} = 0.5$.
10.	$y = 760(0.99988)^x$	Using the formula $y = ab^x$, the initial amount a is 760 mmHg. The pressure decreases by 0.012% for each increase in 1 meter, so the pressure at 1 meter is $100\% - 0.012\% = 99.98\%$ of the pressure at a height of 0 meters. Convert 99.98% to a decimal, and $b = 0.99988$.

Explanations for Chapter 11 Practice Drill 6 - Time Intervals

1.	3 times per day	Since the exponent is *multiplied* by 3, the value is increasing by 18% *multiple* times per day.
2.	2 times per year	Since the exponent is *multiplied* by 2, the value is increasing by 3% *multiple* times per year.
3.	Once per 5 years	Since the exponent is *divided* by 5, the value decreases by 7% once *per* 5 years. When $t = 5$, the exponent is 1.
4.	12 times per year	Since the exponent is *multiplied* by 12, the number of organisms triples *multiple* times per year.

5.	Once per 898 years	Since the exponent is *divided* by 898, the radioactive mass remaining decreases by a half once *per* 898 years. When $t = 898$, the exponent is 1.
6.	$y = 18(2)^{\frac{x}{7}}$	The initial number of goldfish a is 18. Since the number of goldfish doubles every 7 years, $b = 2$. The number of goldfish doubles once *per* seven years, so the exponent needs to be divided by 7.
7.	$y = 40(3)^{4x}$	The initial number of bacteria a is 40. The bacteria are tripling, so $b = 3$. They triple *multiple* times per hour, so we need to multiply the exponent by 4.
8.	$y = 29(1.11)^{\frac{x}{2}}$	The initial value of the stock is \$29, so $a = 29$. The value increases by 11%, so $b = 1.11$. Since it happens once *per* 2 hours, the exponent needs to be divided by 2.
9.	$y = 650,000(0.995)^{3x}$	The initial value of the house is \$650,000, so $a = 650,000$. The value of the house decreases by 0.5%, so the price becomes $100\% - 0.5\% = 99.5\%$ of what it was previously, and $b = 0.995$. Since this happens *multiple* times every year, we need to *multiply* the exponent by 3.
10.	$y = 9\left(\frac{1}{2}\right)^{4x}$	The initial sample is 9 grams, so $a = 9$. It decays by $\frac{1}{2}$, so $b = \frac{1}{2}$. It decays *multiple* times per hour, so we need to multiply the exponent by 4.

Explanations for Chapter 11 Practice Drill 7 - Exponential Graphs

1.	$y = 3^0 = 1$ $y = 3^1 = 3$	Start by finding the y-intercept, where $x = 0$. Now we know the graph has to include the point (0, 1). So it must be either (B) or (D). (It can't be (E) or (F) because it is an increasing exponential.) Now find the point where $x = 1$. So, (1, 3) has to be on the graph, which is answer choice (B).
2.	$y = 2^0 = 1$ $y = 2^1 = 2$	Start by finding the y-intercept, where $x = 0$. Now we know the graph has to include the point (0, 1). So it must be either (B) or (D). (It can't be (E) or (F) because it is an increasing exponential.) Now find the point where $x = 1$. So, (1, 2) has to be on the graph, which is answer choice (D).
3.	$y = 2(3)^0 = 2(1) = 2$	Start by finding the y-intercept, where $x = 0$. Now we know the graph has to include the point (0, 2). So it must be answer choice (C).
4.	$y = 3(2)^0 = 3(1) = 3$	Start by finding the y-intercept, where $x = 0$. Now we know the graph has to include the point (0, 3), and it must be answer choice (A).

5.

$y = 2^{-0} = 1$

$y = 2^{-(-1)} = 2^1 = 2$

Start by finding the y-intercept, where $x = 0$. Now we know the graph has to include the point (0, 1). It's also decreasing because of the negative exponent, so the answer is either (E) or (F).

The easiest point to compare between (E) and (F) is the different values at $x = -1$, so try out -1, which gives us the point $(-1, 2)$ and answer choice (F).

6.

$y = 3^{-0} = 1$

$y = 3^{-(-1)} = 3^1 = 3$

Start by finding the y-intercept, where $x = 0$. Now we know the graph has to include the point (0, 1). It's also decreasing because of the negative exponent, so the answer is either (E) or (F).

The easiest point to compare between (E) and (F) is the different values at $x = -1$, so try out -1, which gives us the point $(-1, 3)$ and answer choice (E).